S6P3 — Exercises from http://www.barsamian.am/mathsexams/

Contents

1 Chapter 1 — Probabilities (S5, Bayes)

Exercise 1 — June 2022 Calc.: ✓

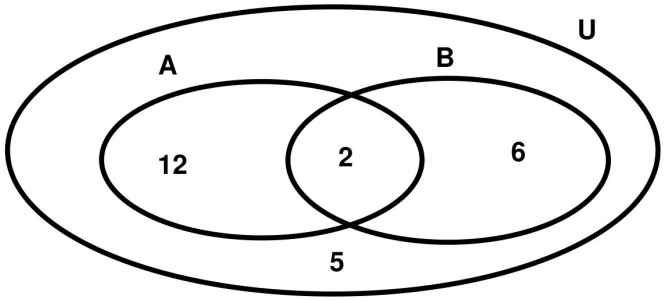
The sensitivity of a Covid test is the probability that someone who is infected with Covid tests positive.
The specificity of a Covid test is the probability that someone who is not infected with Covid tests negative.
Els and Peter use a Covid self-test with a sensitivity of 97% of a specificity of 99%. In the city where Els and Peter live, 1% of the population has Covid.
Use a Venn diagram, a table, or a tree scheme for the following calculations.
Write your answers to the following questions in percent and round to 2 decimal places.

1. Calculate the probability that Els tests positive for Covid. 3
2. Calculate the chance that Els tests positive but does not have Covid. 3
3. Peter tests positive. What is the probability that Peter has Covid? 3
4. The city where Els and Peter live has 100 000 inhabitants. If all residents of this city take a self-test, how many people have a "false positive" test result? 3

Exercise 2 — June 2022 Calc.: ✓

The students of a class are represented in the set U.
Set A is the set of pupils who wear glasses.
Set B is the set of students who have blue eyes.

1. Calculate $P(\mathrm{B})$. 2
2. Calculate $P(\mathrm{A} \cup \mathrm{B})$. 2
3. Calculate $P(\mathrm{A}|\mathrm{B})$. 2
4. Calculate $P(\mathrm{B}|\overline{\mathrm{A}})$. 2
5. A student with blue eyes leaves the classroom. Calculate the probability that this student is wearing glasses. 2



Exercise 3 — June 2022 Calc.: ✓

Consider a rapid test to determine infection with a certain disease. We know that someone who has the disease will be correctly diagnosed 96% of the time. Someone who does not have the disease will be falsely diagnosed, a false positive, 2% of the time.
The disease is prevalent in 0.4% of the population.

1. A patient takes a test. Calculate the probability that they will get a positive result. 3
2. The test result comes back positive. Calculate the probability that the patient has the disease given that they got a positive result. 4

Exercise 4 — June 2022 Calc.: ✗

In an ice cream parlor you can choose from 2 flavors of ice cream: chocolate or vanilla. A combination of flavors is not allowed. You can get the ice cream in a cone or a cup.
In this ice cream parlor, 50% of the customers choose a cone and 50% opt for a cup.
35% of customers choose a cup with chocolate ice cream.
20% of customers take vanilla ice cream.

1. A new customer enters the ice cream parlor. Calculate the probability that the customer chooses a cone with vanilla ice cream. 4
2. The next customer chooses vanilla ice cream. Calculate the probability that this customer wants a cone. 4
3. Are the events "choosing a cone" and "choosing chocolade ice cream" independent events? Explain your answer. 4

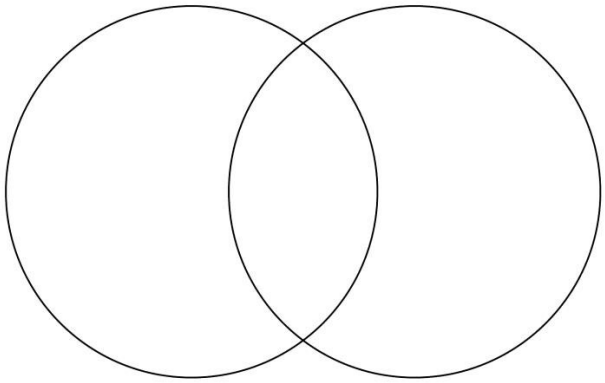
Exercise 5 — December 2021 Calc.: ✓

110 participants are taking part in a day-long conference. There are two different lectures being given on two different themes; politics and the economy. 62 sign up for the politics lecture. There are 51 participants who will go to the politics lecture and the economy lecture. 32 people will attend neither lecture.

1. Use the following sets to complete the following Venn diagram: 3

   P: The participant is attending the politics lecture.

   E: The participant is attending the economy lecture.



2. A person is chosen at random from the crowd to do an interview. What is the probability that this person: 3
   (a) Will have attended the politics lecture or the economy lecture?
   (b) Will have attended the economics lecture, but not the politics lecture

Exercise 6 — December 2021 Calc.: ✗

In a box of chocolates, we find 24 different chocolates. 18 chocolates are made from milk chocolate and 6 are made from white chocolate. Two thirds of the milk chocolates have a marzipan filling. In total there are 16 chocolates with a marzipan filling in the box.

1. Complete the following two-way table. 5

Marzipan \ Chocolate	Milk Chocolate	White Chocolate	Total
With			
Without			
Total			

2. If a chocolate is picked at random from the full box, calculate the probability that it would be a white chocolate one without a marzipan filling. 2

3. Given that a chocolate chosen at random from the full box is a white chocolate, calculate the probability that it has a marzipan filling. 2

Exercise 7 — December 2021 Calc.: ✗

In a box of 4 matches one is shorter than the others. Four players pick a match one after the other. Whoever picks the short match loses.

1. Show, with the aid of a tree diagram the probabilities of each player getting the short match. 4

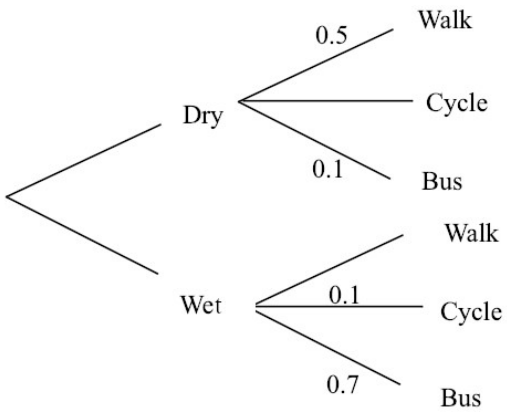
2. Give the following probabilities: 4
   - The first player loses:
   - The second player loses:
   - The third player loses:
   - The fourth player loses:

3. Does it have an effect on the outcome whether you are the first to choose the match or the last? 2

Exercise 8 — June 2019 Calc.: ✓

Andy can walk to work, cycle or travel by bus. The choice he makes depends on the weather.

1. Copy and complete the tree diagram to show the probabilities for each of Andy's methods of travel. 2



2. A day is selected at random. Calculate the probability of the following events:
   (a) The weather is wet and Andy travels by bus. 2
   (b) Andy walks or cycles. 3
   (c) The weather is dry given that Andy walks or cycles. 3

Exercise 9 — June 2019 Calc.: ✗

A company sells two printers with an extended warranty; the Grafter and the Elite.
For the first 50 sales of each type of printer the number of claims are recorded as follows:

Printer \ Claim	Yes	No
Grafter	15	35
Elite	10	40

A purchaser is selected at random from this group. 5

1. What is the probability that they made a claim on their printer?
2. Given that they didn't make a claim what is the probability that they bought the Elite?

Exercise 10 — June 2019 Calc.: ✗

A group of 30 students went on a camping trip.

1. Of these, 12 returned with both sunburns and insect bites and 20 reported sunburn. How many suffered only insect bites if it is known that three students suffered neither? Draw a **Venn-Diagram** to illustrate the situation. 5
2. In the group, 9 students had food allergies. Of the 16 girls in the group 5 had food allergies. A student from the group is picked at random. What is he probability that they don't have food allergies given that they are a boy? Draw a **two-way table** to illustrate the situation. 6

Exercise 11 — June 2017 Calc.: ✓

The workforce in a Mickey Mouse factory has to dress up for a fancy dress one day:

Costume \ Sex	Male	Female
Mickey Mouse	10	5
Minnie Mouse	2	12
Pluto	8	3

What is the probability that a member of the workforce is:

1. Minnie Mouse; 2
2. Pluto and male; 2
3. Male, given that they are dressed as Minnie Mouse. 2

Give your answers to 2 decimal places.

Exercise 12 — June 2017 Calc.: ✗

A single unbiased die has its faces labelled 1, 1, 2, 2, 3, 4.
A player throws the die twice and adds up the numbers to get a final score.
Use a 2-dimensional grid, or any other suitable way, to solve the following:

1. Calculate the probability that the final score is 3. 2
2. Given that the 1st time the die was thrown it was even, calculate the probability that the final score will be even. 3

2 Chapter 2 — Functions (S5)

Exercise 13 — June 2022 Calc.: ✓

In a village with 700 inhabitants, 14 of them decide to start a rumour at the same time.
After 15 hours the rumour has been heard by all of the inhabitants.
A linear function is proposed to model this problem.

1. **Explain**, why the function 5

$$f(t) = 45,73 \cdot t + 14$$

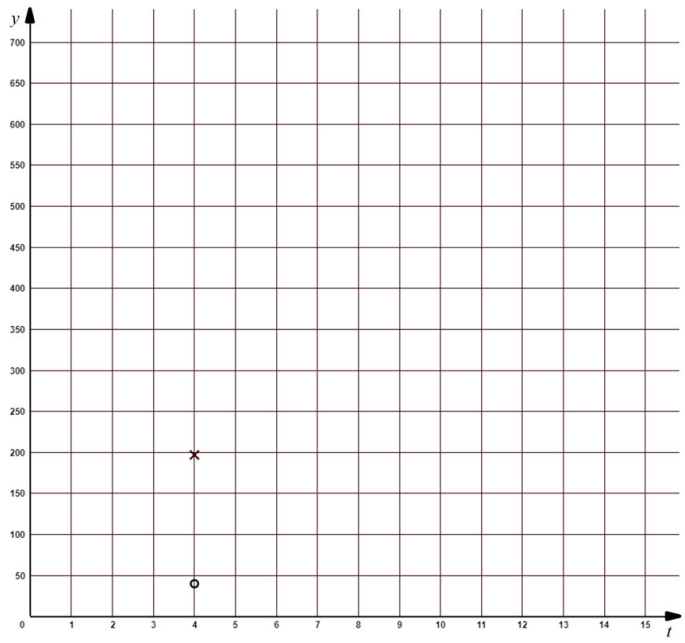
could be used to model this problem, what the variables f and t represent with units, and what the numbers represent.

2. **Determine** the domain of the function. 2

3. Use this function to **calculate** the time taken for half of the inhabitants to have heard the rumour. 3

4. **Copy** the graph below onto your 5 mm square answer paper using a scale of 1 cm for 1 unit on the horizontal axis and 1 cm for 50 units on the vertical axis. 3

Draw the line representing the function f on your copy of the graph. One of the points has already been marked for you with an X (the point marked O is used later in the question).



(This question continues on the next page)

Another function is now proposed to model this problem

$$g(t) = 14 \cdot 1,298^t.$$

5. **Givee** the name of the type of model represented by function g. 1

6. **Draw** the line representing the function g on the same graph as for f above. One of the points has already been marked for you with an O. 3

7. Using your graph or otherwise, **determine** also for this function the time taken for half of the inhabitants to have heard the rumour. 3

8. **Compare** the two functions f and g and **decide**, with a reason, which is the better model for this situation. 4

Exercise 14 — December 2021 Calc.: ✓

A group of scientists decides to investigate a population of insects in a large field. It is found that the starting population 100 and that the population increases exponentially by 20% every week.
Two students each write down a formula to model the population P at a time t, where t is the number of days since the start of the investigation:
Formula A: $P(t) = 100t + 1.2$
Formula B: $P(t) = 100 \cdot (1.2)^t$

1. Explain why formula B is the correct formula and why formula A is incorrect. 2
2. Calculate the number of insects after 2 weeks, to the nearest whole number. 2
3. Copy and complete the table of values below, giving your answers to the nearest whole number: 2

Number of days	**5**	**10**	**15**	**20**
Population				

4. After how many days will the population exceed 4 600? 2

Another group of scientists investigates a population of insects in a different large field. They record their results in the table below:

Number of days	**0**	**5**	**10**	**15**	**20**
Population	**100**	**340**	**580**	**820**	**1 060**

5. Explain why the results follow a **linear** model. 1
6. Use the information in the table of values to write down a formula to model the population P at a time t , where t is the number of days since the start of the investigation. 2

Exercise 15 — December 2021 Calc.: ✓

Consider the function $f(x) = \dfrac{6x+5}{3x-4}$.

1. Explain why the function is undefined when $x = 1\dfrac{1}{3}$. 1
2. State the domain of the function. 2
3. Give the coordinates of the y-intercept of $f(x)$. 2

Exercise 16 — December 2021 Calc.: ✓

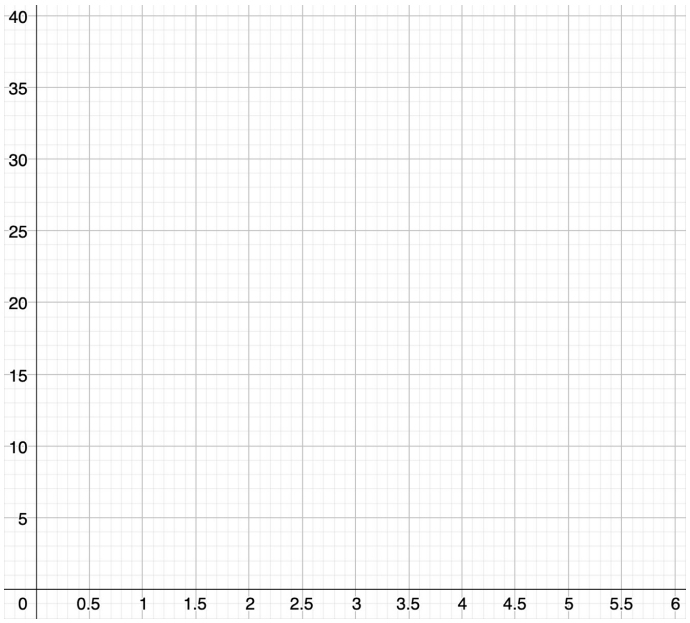
A water bomb is catapulted in the air. The height h, in metres, after t seconds is given by the function

$$h(t) = -4.9t^2 + 27t + 2.4$$

1. Complete the following table: 2

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5
$h(t)$												

2. Draw the graph which represents the trajectory of the water bomb on the following cartesian plane: 2



3. What height is the water bomb after 1 second? 1

4. Estimate the maximum height achieved by the water bomb? Round to the nearest metre. 1

5. How long does the water bomb stay above 30 metres high? You may answer with a calculation or a graphic interpretation. Round to nearest 0.1 seconds. 2

6. Solve the equation $-4.9t^2 + 27t + 2.4 = 0$. After how much time will the water bomb explode on the ground? Round the answer to the nearest 0.1 second. 2

Exercise 17 — December 2021 Calc.: ✓

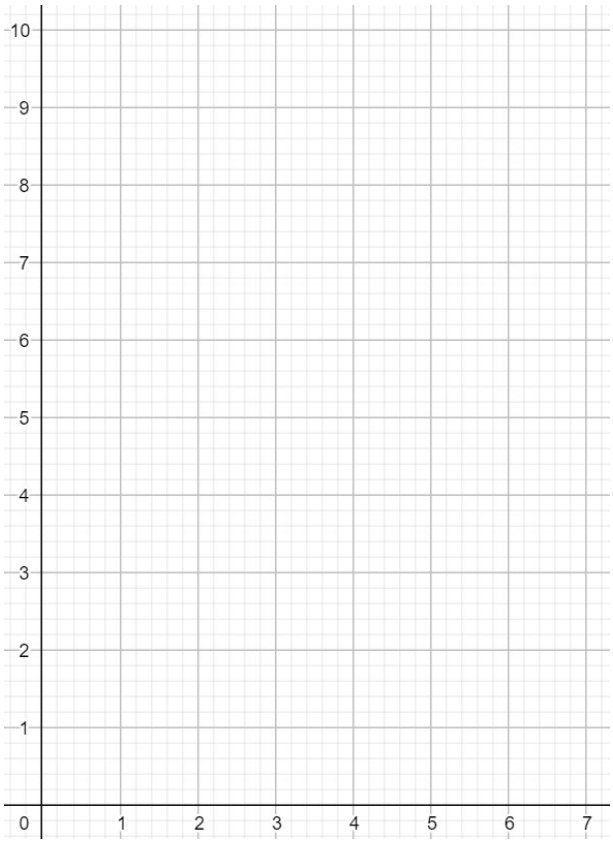
A population of bacteria in a petri dish has a growth model given by the function

$$f(x) = 3^{x-4}$$

Where x represents the number of days passed and $f(x)$ represents the number of thousands of bacteria present.

1. Using the grid below, draw a graph to represent the population of the bacteria over a week long period. Use the following table of values if you wish. 4

x	0	1	2	3	4	5	6	7
$f(x)$								



2. Calculate the number of bacteria in the dish at the beginning of the observation. 2

3. Use your graph to estimate the amount of bacteria present on day 6. 2

4. Use your graph to estimate to the nearest day, how long it will take to pass 8000 bacteria. 2

Exercise 18 — December 2021 Calc.: ✗

The following image is a graphic representation of the function

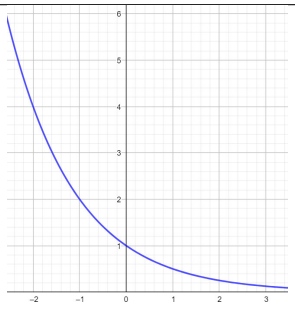
$$f(x) = 2^{3x+1} + 2.$$

1. State the domain of the function. 2
2. State the range of the function. 2
3. Find the y-intercept. 2
4. What are the roots of the function? 2
5. Estimate the following values: 6
   (a) $f(0.2) = \ldots$
   (b) $f(-2) = \ldots$
   (c) if $f(x) = 3$, then $x = \ldots$

Exercise 19 — December 2021 Calc.: ✗

Match each of the following functions to their corresponding graph. 8

Function	$f(x) = 2^{2x} + 3$	$g(x) = 3^{-x}$	$h(x) = 0.5^{x}$	$q(x) = 2^{x}$
Graph				



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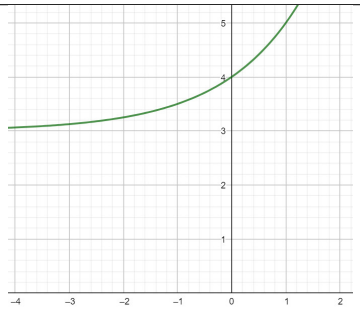

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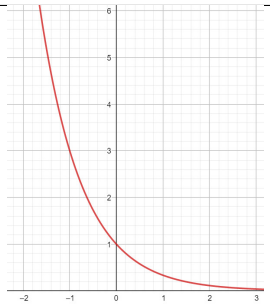

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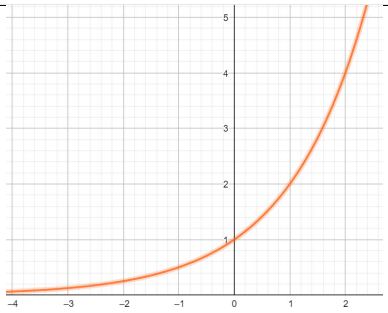

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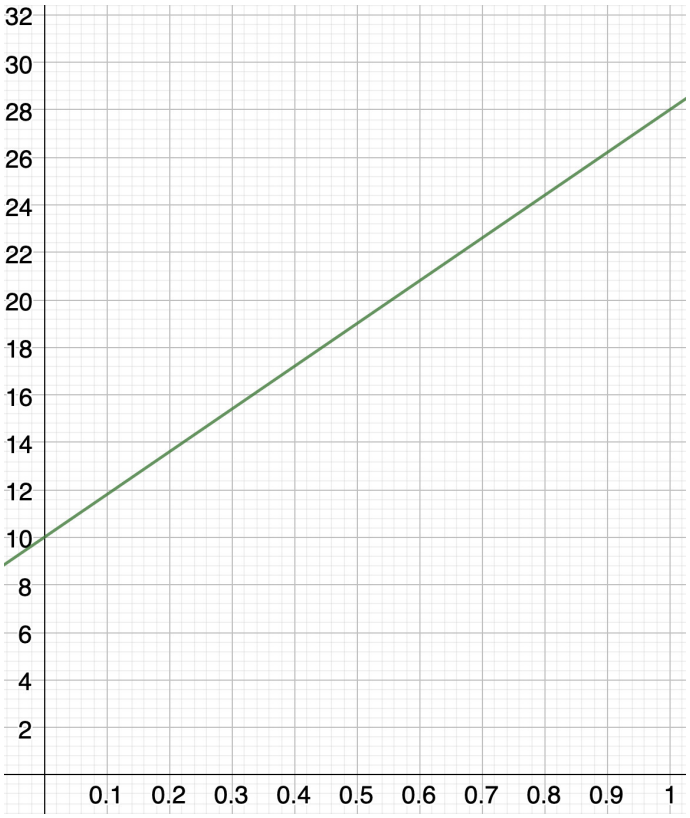
Exercise 20 — December 2021 Calc.: ✗

Helen is taking part in a cycling race. She has already cycled 10 km and is advancing at a constant speed. The following graph represents the distance travelled as a function of the time in hours.

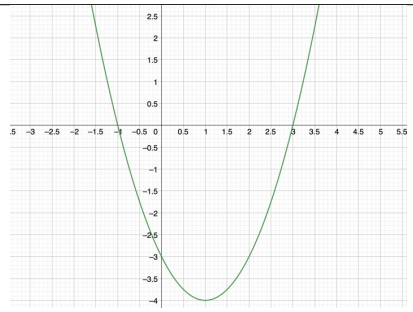


1. Identify the distance travelled at the origin (of the graph) and the slope of the line. At what speed is Helen travelling? 4
2. Formulate an equation for the distance, d (in km) that Helen cycles as a function of time, t (in h) since she passed the 10 km mark. 2
3. How many kilometres will Helen have cycled, 90 minutes after passing the 10 km mark? 3

Exercise 21 — December 2021 Calc.: ✗

Match each of the following functions to their corresponding graph. 8

Function	$A(x) = -2x^2 + 3x + 3$	$B(x) = (x-3)(x+1)$	$C(x) = -2(x+4)^2 + 5$	$D(x) = (x+3)^2 - 1$
Graph				



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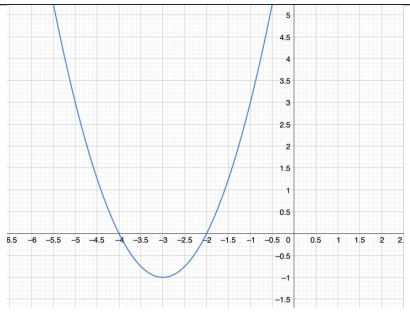

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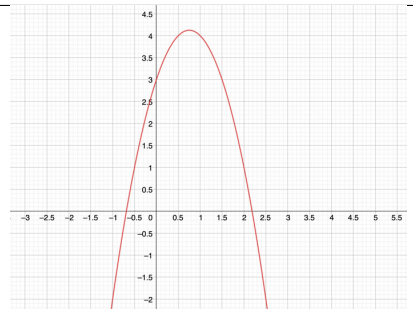

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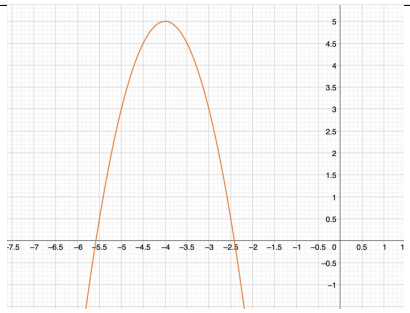

d

Exercise 22 — December 2018 Calc.: ✓

A volleyball player serves from the back line of the court to send the ball into the adversary camp. The height h of the ball, in meters, is given by the following function :
$h(t) = -4.9t^2 + 3.8t + 1.7$, where t is in seconds.
(For this exercise give all answers correct to 2 d.p.)

1. What is the maximum height reached by the ball? 3
2. After how long will the ball fall to the ground? 3
3. For how long does the ball stay above 1.5 m ? 3
4. The ball will reach the net at $t = 0.6$ s. The height of the net is 2.34 m.
   Will the ball pass over the net into the adversary camp? Explain. 3

Exercise 23 — June 2017 Calc.: ✗

Sketch the graph of the parabola $y = x^2 - 2x - 8$. 7
Your sketch must show the coordinates for any points of intersection with the coordinate axes and the coordinates of the vertex.

3 Chapter 3 — Random variables

Exercise 24 — June 2022 Calc.: ✓

When you play roulette at a casino, we can bet on one of the 37 numbers from 0 to 36. Alternatively you can bet on the colour red or black (0 is coloured green).
For the remainder of the question, consider a bet of €100.

1. If you bet on red and the ball falls on one of the 18 red compartments, your money is doubled.
   Calculate the Expected value of the random variable X: « amount won betting on red ». 3
2. Compare this value with the Expected value of the random variable Y: « Amount won betting on a specific number ». If the ball stops in the compartment you have bet on, you receive 36 times what you bet. 4

Exercise 25 — June 2022 Calc.: ✓

In a basket are 5 white and 3 red socks. You take two random socks out of the basket.

1. Draw a tree diagram for this experiment and write down the probabilities for each branch of your tree diagram. 4

The stochastic variable X is "the number of red socks".

2. Give the probability distribution of X in a table. 6
3. Calculate the expected value of X. Write down all steps in your calculation. 4

Exercise 26 — June 2022 Calc.: ✗

The probability distribution of a stochastic variable X is given.

x	0	1	2	3	4
$P(X = x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

1. Explain why this table is a probability distribution. 2
2. Calculate the expected value of X. 2
3. Calculate $P(X > 2)$ 2
4. Calculate $P(X < 4)$ 2

4 Chapter 4 — Derivatives

Exercise 27 — June 2022 Calc.: ✓

Below are three functions labelled A, B, C, D and E and their derivatives labelled I, II, III, IV and V.
Match each function to its corresponding derivative.

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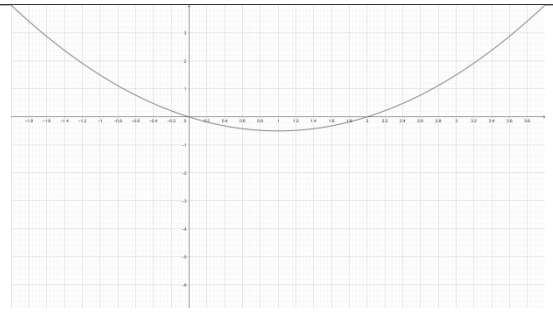

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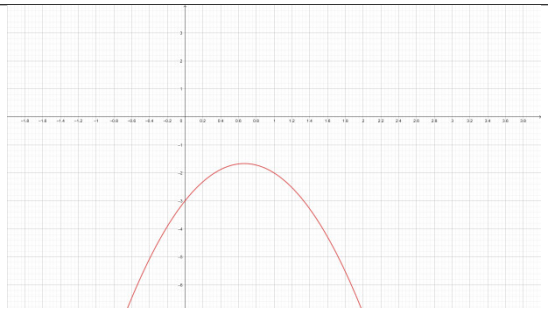

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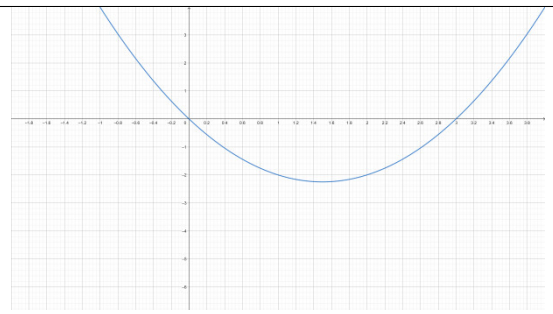

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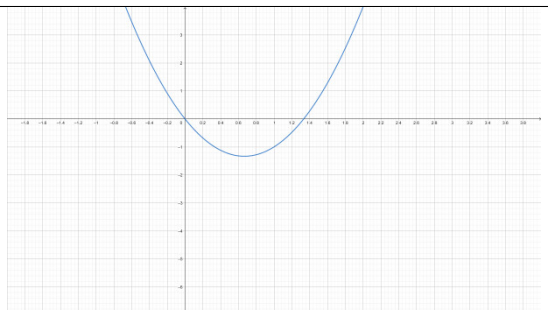

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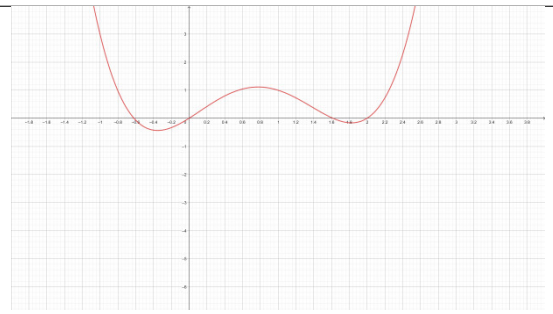

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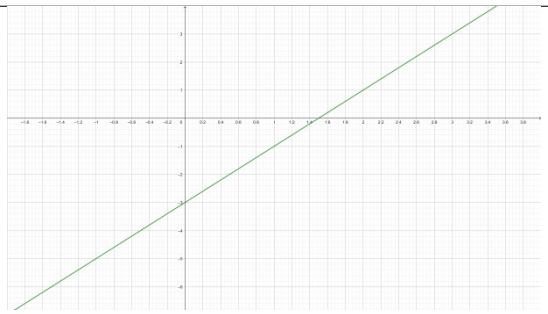

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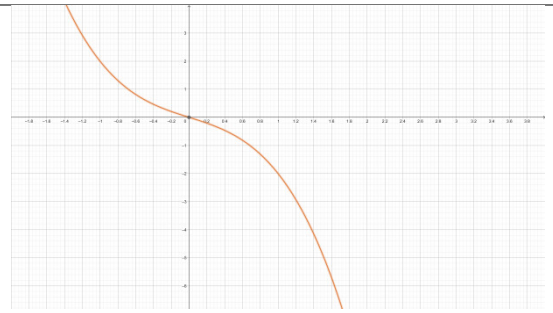

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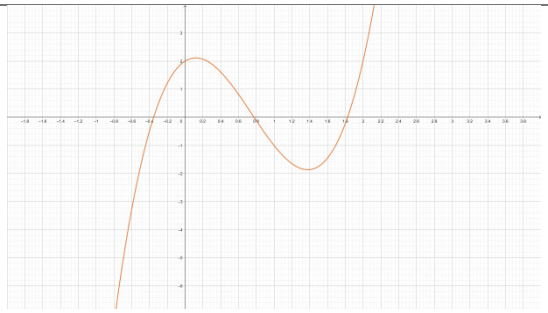

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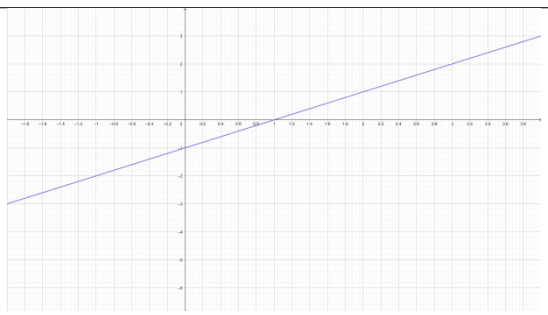
E



V

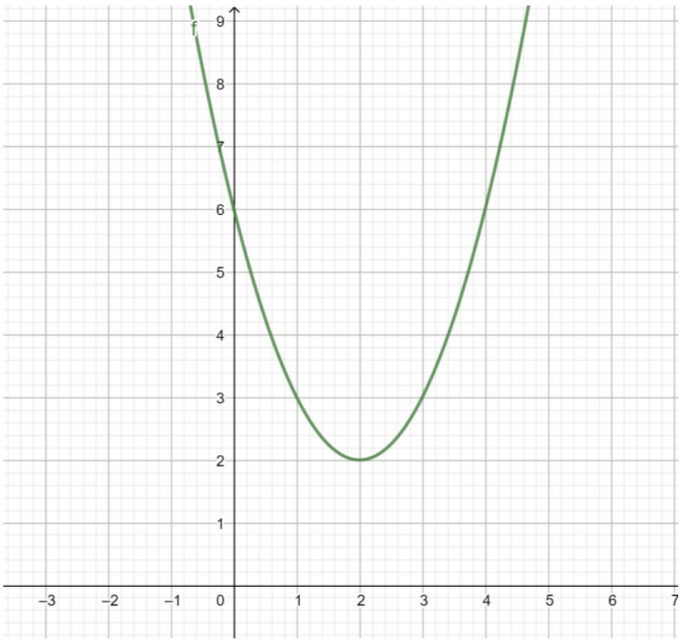
Exercise 28 — June 2022 Calc.: ✓

Consider the function f defined by $f(x) = -\frac{x^3}{3} - \frac{x^2}{2} + 6x + 4$.

1. Determine an expression for f', the derivative of the function f. 4
2. Study the sign of f' (make a sign table). 6
3. Determine the interval in which the function f is increasing. 4

Exercise 29 — June 2022 Calc.: ✗

1. Find the equation of the tangent line to the function f at the point $(1, 3)$, given: : $f(x) = x^2 - 4x + 6$. 5
2. Accurately draw the tangent to the function on the accompanying graph. 2



Exercise 30 — December 2021 Calc.: ✓

Karen plays volleyball and throws a ball vertically. The height $h(t)$ (in meters) as a function of the time t (in second) of the ball is given by the formula: $h(t) = 6t - 5t^2 + 2$.

1. From what height does Karen throw the ball? 2
2. Show that the ball reaches its highest point at $t = 0.6$ s. 3
3. Calculate the ball's maximum height. 3
4. For how long is the ball in the air? 3

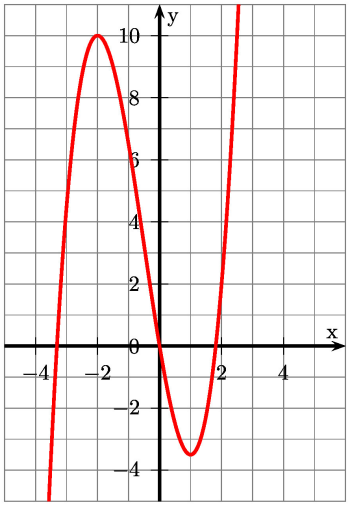
Exercise 31 — December 2021 Calc.: ✓

Consider the function $f(x) = x^3 - 4x^2 + x + 2$.

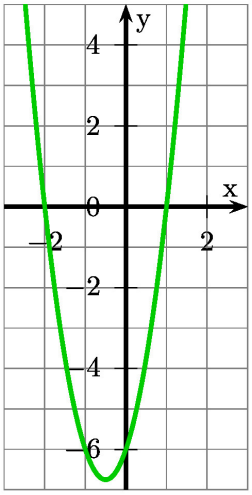
1. Determine the coordinates of the turning points of $f(x)$, giving your answer to 2 decimal places. 4
2. Draw a table of signs. 2
3. Use the table of signs to determine the nature of the turning points. 2

Exercise 32 — December 2021 Calc.: ✗

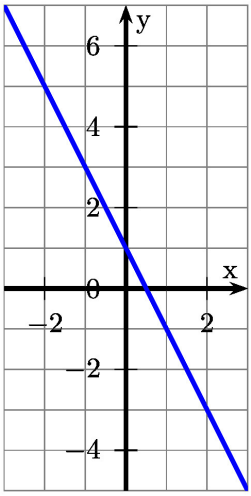
The graph below is the graph of the function f.



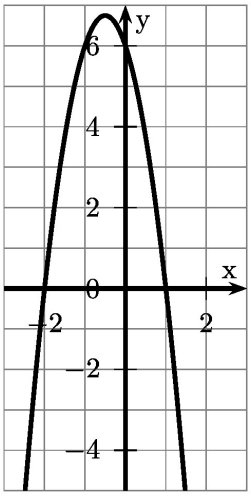
Which of the 4 graphs below is the corresponding graph of f'? 4
For each graph you **must** explain why it is or is not the correct graph.



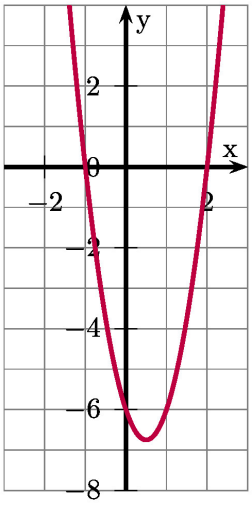
Graph A



Graph B



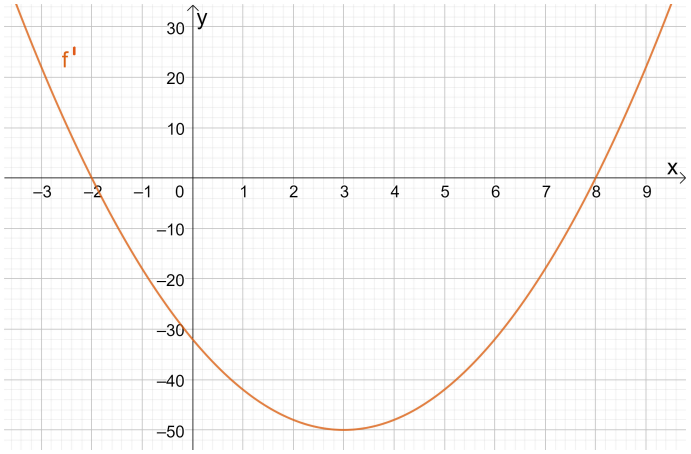
Graph C



Graph D

Exercise 33 — December 2021 Calc.: ✗

The graph of the derivative $f'(x)$ is given below.



1. Give the x-coordinates of the two turning points. 2
2. For which values of x is the graph of $f(x)$ increasing? 2
3. For which value of x does $f(x)$ reach a minimum? 2
4. Sketch a possible graph of $f(x)$, given that the point $(8, 0)$ lies on the graph of $f(x)$. 3

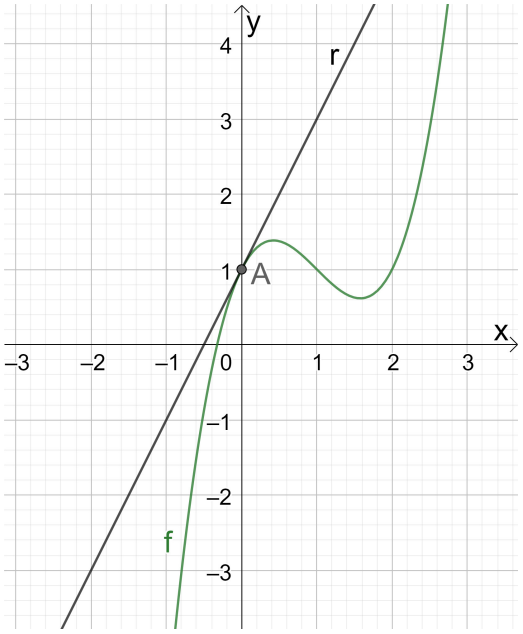
Exercise 34 — December 2021 Calc.: ✗

The function f is defined as $f(x) = 2x^2 - 8x + 8$.

1. Determine the coordinates of the y-intercept. 2
2. Calculate $f(2)$ 2
3. Determine the derivative $f'(x)$. 2
4. For what value of x does the function $f(x)$ have a turning point? State the nature of the turning point and explain your answer. 3
5. Find the equation of the tangent to the curve at the point $(1, 2)$. 4
6. The point A is a point on the graph of f. The gradient at the point A is equal to 12.
   Find the coordinates of the point A. 4

Exercise 35 — December 2021 Calc.: ✗

Consider the graph of the function f shown below. The line r is a tangent line to the graph of f at point A.



1. Use the information in the diagram to find the equation of the line r. 4
2. Given that $f(x) = x^3 - 3x^2 + 2x + 1$, use the diagram or otherwise to find the value of $f'(0)$. 4

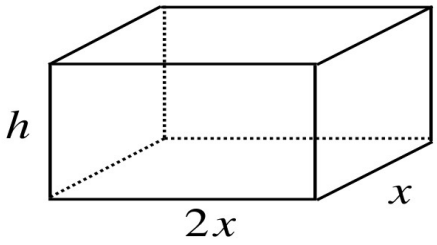
Exercise 36 — December 2021 Calc.: ✗

Give the derivative $f'(x)$ of the following functions:

1. $f(x) = x^3 - 3x^2$ 2
2. $f(x) = 2x^2 + x - 3$ 2
3. $f(x) = \frac{1}{2}x - \frac{1}{3}x^3 + \frac{2}{3}x^6$ 2

Exercise 37 — June 2019 Calc.: ✓

The length of the base of a cuboid is twice the width x, and its height is h centimetres, as shown in the diagram below.
Its total surface area is A cm^2 and its volume is V cm^3.



1. Show that $A = 4x^2 + 6xh$. 2

The manufacturer needs the total surface area $A = 300$ cm^2.

2. Find an expression for h, in terms of x. 2
3. Show that the volume $V(x) = 100x - \frac{4}{3}x^3$. 1
4. Determine the maximum volume possible for the cuboid and determine the value of h that achieves this. 5
   You should explain in your answer how you know that this will be a maximum value.

Exercise 38 — June 2019 Calc.: ✓

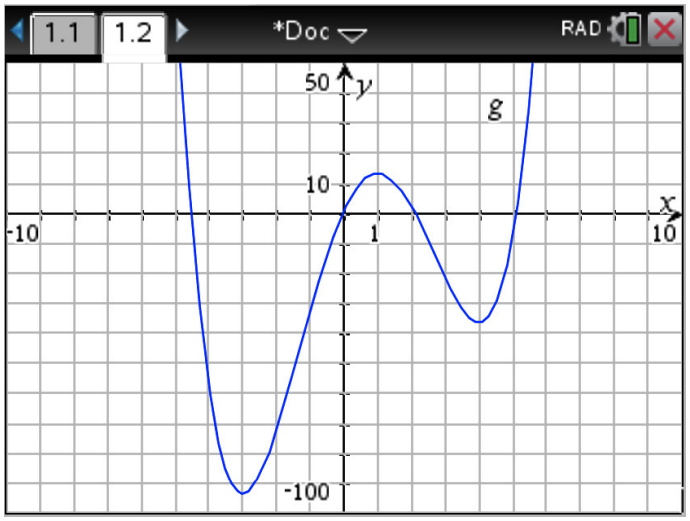
The function $y = 2x^3 - 5x^2 - 4x + 2$ is defined for $x \in \mathbb{R}$.

1. Use differentiation to determine the (x, y) coordinate for any stationary points of the function y. 2
2. Classify the nature of any stationary points in terms of local maxima or minima. 3
3. Find the range of x values for which the curve is increasing. 2
4. Find the equation of the tangent line at $x = 1$. 3

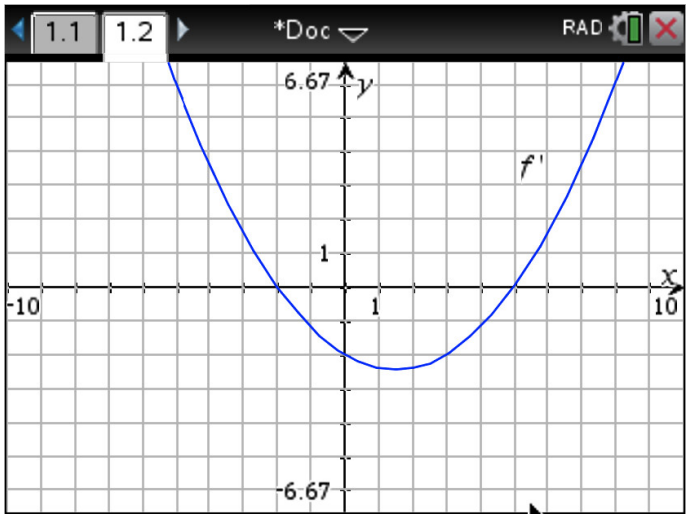
Exercise 39 — December 2018 Calc.: ✓

ANSWER ON THIS SHEET AND RETURN WITH OTHER ANSWER SHEETS

1. The figure represents the graph of a function $g(x)$. Sketch a possible graph for the function $g'(x)$ on the same grid. 4



2. The figure represents the graph of a derivate function $f'(x)$. Sketch a possible graph for the function $f(x)$ on the same grid. 4



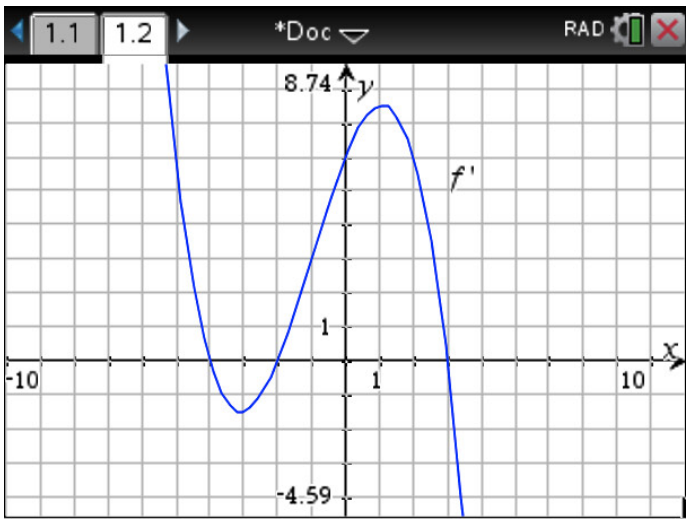
Exercise 40 — December 2018 Calc.: ✓

Consider the function $f(x) = -x^3 - 3x^2 + 5x + 7$ and its graph F.

1. Draw a table of signs showing the variations of function f. 6
2. Find the coordinates of the turning points of F and state their nature. Give answers correct to 1 d.p. 2
3. Find the equation of the tangent to the graph at $x = -1$. 2
4. Find the coordinates of the points on F where the tangent has slope 5. 2
5. Find the equation of the tangents to F with slope 5. 2

Exercise 41 — December 2018 Calc.: ✓

The figure represents the graph of a derivate function f' of a function f.

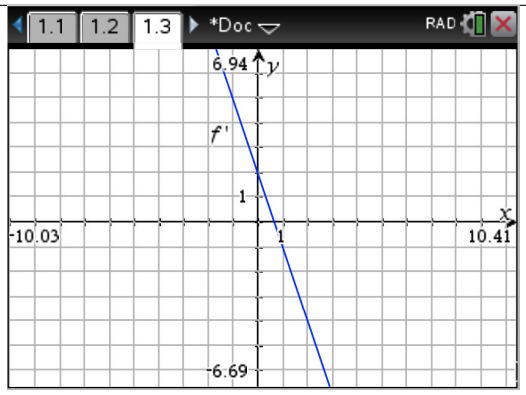


1. Give the x-coordinates of the extrema of f and state their nature. 6
2. From the graph of f' find the slope of the tangent to the graph of f at $x = -1$. 2
3. Find the solutions for $f'(x) = 6$. 3
4. The graph of function f passes through point P(0, 1). Find the equation of the tangent to the graph of f at point P. 4

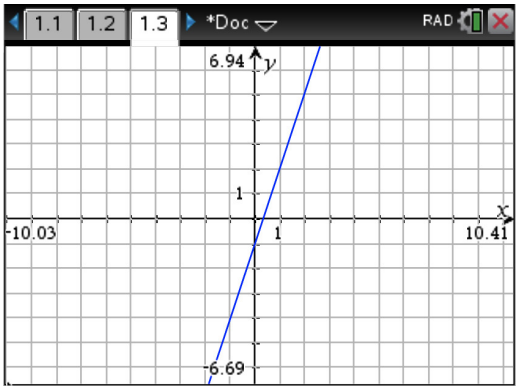
Exercise 42 — December 2018 Calc.: ✗

The figure on the right represents the graph of a derivate function f'.
Choose among the graphs below the one(s) that could represent the function f.
You must justify your answer carefuly, otherwise no points will be awarded.

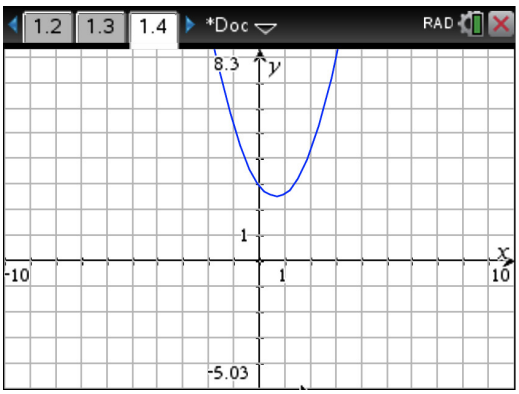
6



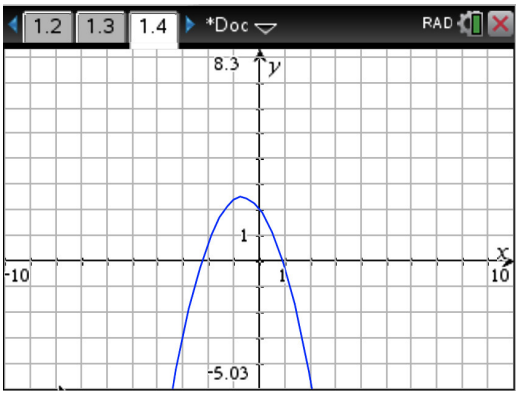
Graph of function f'



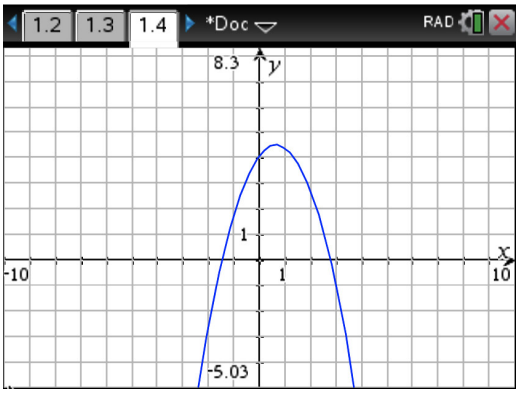
Graph 1



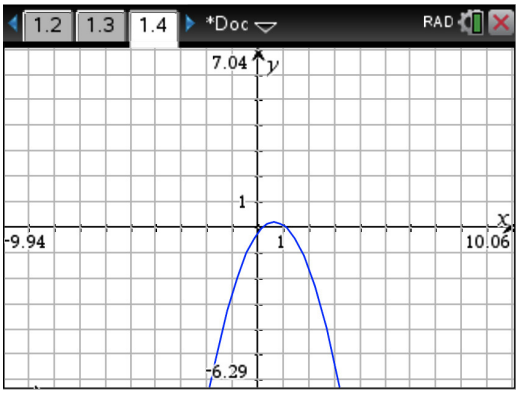
Graph 2



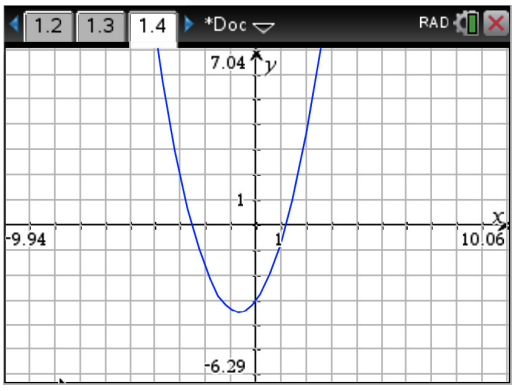
Graph 3



Graph 4



Graph 5



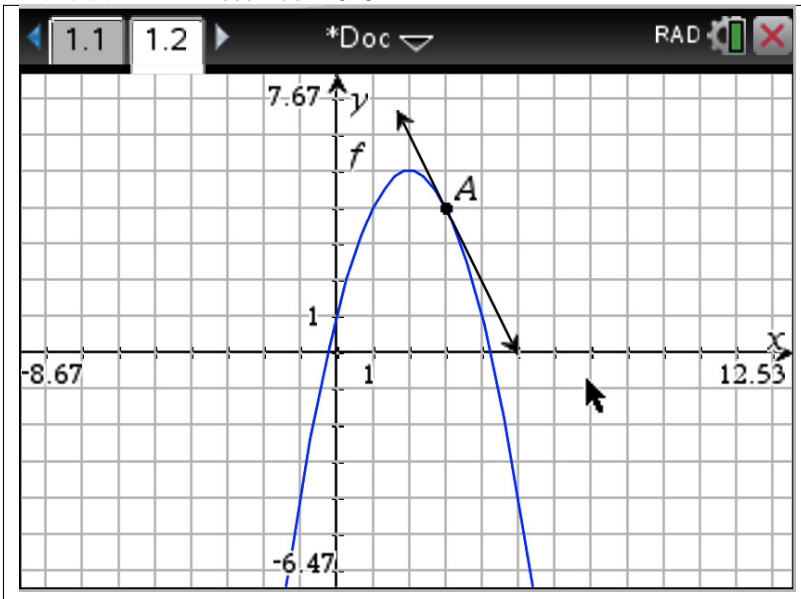
Graph 6

Exercise 43 — December 2018 Calc.: ✗

Consider the function $f(x) = x^2 - 2x - 8$ and its graph F.

1. Find the coordinates of the turning point of F. 2
2. Write the equation of the tangent to F at $x = 2$. 4
3. Find the coordinates of the intersection point of F with the line $y = -x - 2$. 4

Exercise 44 — December 2018 Calc.: ✗



The figure shows the graph of function f.

1. From the graph find the values of $f(0)$, $f(2)$ and $f(3)$. 3
2. From the graph find the values of $f'(2)$ and $f'(3)$. 4
3. Write the equation of the tangent to the graph at point A. 4
4. From the graph find the range of values for x such that $f'(x) < 0$. 4

Exercise 45 — December 2018 Calc.: ✗

Differentiate the following functions.

1. $f(x) = -3x^3 + 6x^2 - \frac{13}{217}$ 2
2. $g(x) = \frac{1}{2}x^4 - \frac{1}{3}x^3$ 2

Exercise 46 — June 2017 Calc.: ✓

The function $f(x) = x^4 + 4x^3 - 4x^2 - 16x$ is defined for $x \in \mathbb{R}$.

1. Use differentiation to determine the (x, y) coordinate for any stationary points of the function f. 2
2. Classify the nature of any stationary points in terms of local maximum or minimum. 3
3. Find the range of values for which the curve is increasing. 3
4. Find the equation of the tangent at $x = 1$. 2

Exercise 47 — June 2017 Calc.: ✗

Find the x-coordinates for the stationary points of the function 5

$$y = x^3 + x^2 - 5x - 6$$

And determine whether or not a stationary point is a local minimum or maximum.
Note : There is no need to calculate the value of the y coordinate in this question.

5 Chapter 5 — Combinatorics

Exercise 48 — June 2022 Calc.: ✓

From a group of 10 runners and 15 non-runners, a university researcher selects 5 people for a study on cardio-vascular disease.	
1. How many groups is it possible to make if we make no distinction between the runners and the non-runners?	3
2. How many groups is it possible to make if we want exactly three runners participating in the study?	3
3. What is the probability that, given a random selection of participants in the study, that there would be exactly three runners in the group?	4

Exercise 49 — June 2022 Calc.: ✗

Six sprinters compete against each other in a final. How many different arrangements could we have on the podium? (The podium consists of a gold medal winner, a silver medal winner and a bronze medal winner).	4

Exercise 50 — June 2022 Calc.: ✓

A box contains letter blocks (see figure).
This box contains the letter blocks C, A, T, M and S.
Els takes 3 random blocks out of the box.

1. Calculate the probability that she can form the word MAT with these three blocks. — 4

Peter takes 3 random cubes out of the box.

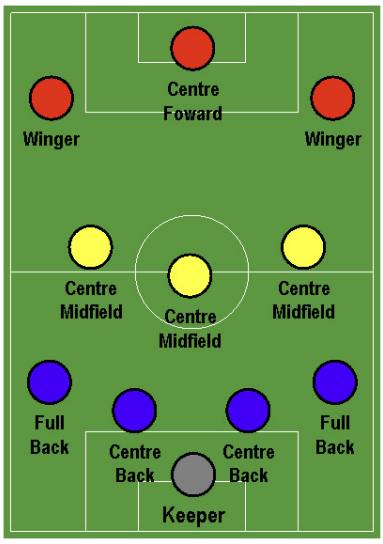
2. The first block he takes is the letter M. Then he takes 2 more letter blocks. Calculate the probability that Peter can form the word MAT knowing that his first letter block is the letter M. — 4

Exercise 51 — June 2022 Calc.: ✓

The coach of a football team selected 24 players for a tournament. He picked 8 defenders, 7 midfielders, 5 strikers and 4 goalkeepers.

1. How many different teams can the coach put together if he chooses a line-up consisting of 1 goalkeeper, 4 defenders, 3 midfielders and 3 strikers? 4

2. The coach has chosen 4 defenders in his line-up: Virgil, Sergio, Ruben and Trent. The trainer can line up these defenders in the places indicated by the blue dots. How many lineups are possible with these 4 defenders? 3



3. For a press conference, a group of 4 players is randomly chosen from all 24 players. Calculate the probability that this group consists of 1 defender, 1 midfielder, 1 striker and 1 goalkeeper. 3

4. The probability that Christiano scores a penalty is 85%. Christiano kicks 5 penalties.

   (a) Calculate the probability that Christiano scores 5 times. 2

   (b) Calculate the probability that Christiano scores 3 out of 5 attempts. 2

   (c) Calculate the probability that Christiano scores 4 times at most. 2

Exercise 52 — June 2022 Calc.: ✗

A class consists of 6 Flemish and 3 Dutch pupils. In this class we select a team of 3 students for the student council.

1. How many different teams of 3 students can be formed? 3

2. How many different teams of 3 students can be formed if each team has at least 1 Flemish and 1 Dutch representative? 3

Exercise 53 — June 2022 Calc.: ✗

The PIN code of a bank card consists of 5 digits.

1. How many different PINs can you create? 3

2. Lisa has a PIN code that consists of 5 digits. Unfortunately, she forgot her PIN. She remembers that her PIN code begins with the number 418 and she also remembers that the numbers 0 and 9 do not appear in her PIN code. How many PIN codes are still possible? 4

Exercise 54 — June 2022 Calc.: ✗

Calculate:

1. $\binom{5}{3}$ 1

2. $\binom{201}{1}$ 1

Exercise 55 — June 2019 Calc.: ✗

Simon has a 4 digit PIN for his phone but he has forgotten what it is. He knows that all the digits are different and that zero is not used. What is the probability that he will find the correct PIN by randomly guessing the digits?	5

Exercise 56 — June 2019 Calc.: ✓

Professor Fry and 11 colleagues from his team went to a restaurant to commemorate the success of the research conducted on Snake Island. At the end of the meal, they decided to randomly select three members of the group to pay the bill.

1. In how many ways can these three people be selected? 3
2. What is the probability that Professor Fry will **not** have to contribute to the payment? 3
3. In the group there are 3 biologists, 5 zoologists and 4 veterinarians. What is the probability that exactly one of each profession will be selected? 3

Exercise 57 — June 2019 Calc.: ✗

A lock consists of three wheels with the digits 0 to 9.

1. Knowing that each digit has only been used once, what is the maximum number of attempts that must be made before the lock will open? 4
2. What is the probability that the lock will open on the first try? 2

6 Chapter 6 — Periodic functions

Exercise 58 — June 2022 Calc.: ✓

The function $f(x) = 60\sin\left(\frac{2\pi}{30}(t-7,5)\right) + 75$ can be used to model the height of a passenger above the ground on the London Eye, where t is the time in minutes after departure.

1. Determine the period of the London Eye. 2
2. Determine the amplitude of the London Eye. 3
3. Use this function to estimate the height of a passenger 18 minutes after departure. 2
4. What height above the ground is the boarding platform? 3
5. Sketch the graph of the function $f(x)$. 4
6. Use your graph to estimate how long a passenger would spend more than 100 m above the ground. 3

Exercise 59 — June 2022 Calc.: ✗

The following data set can be modelled by the function:

$$f(x) = a\sin(b(x-c)) + d$$

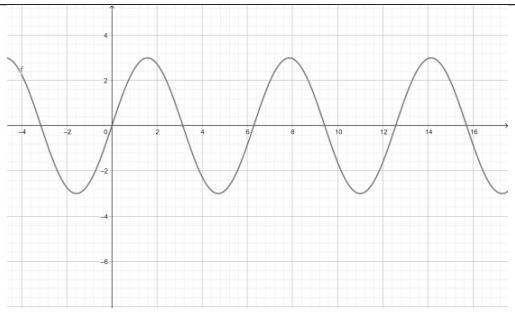
x	2	3	4	5	6	7	8	9	10	11
y	11	9.8	7	4.2	3	4.2	7	9.8	11	9.8

1. Estimate the amplitude of the function. 1
2. Estimate the period of the function. 1
3. Estimate the vertical translation of the function. 1
4. Estimate the horizontal translation of the function. 1
5. Fill in the appropriate values of a, b, c and d to write the cosine function which models the data. 3

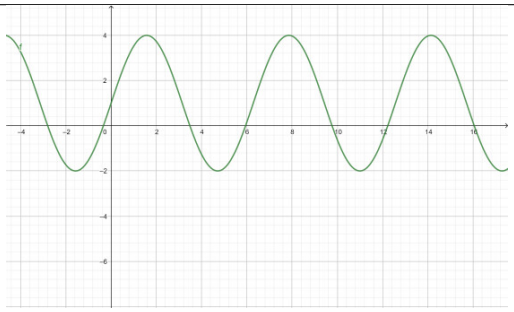
Exercise 60 — June 2022 Calc.: ✗

Match each of the following functions to their corresponding graph: 8

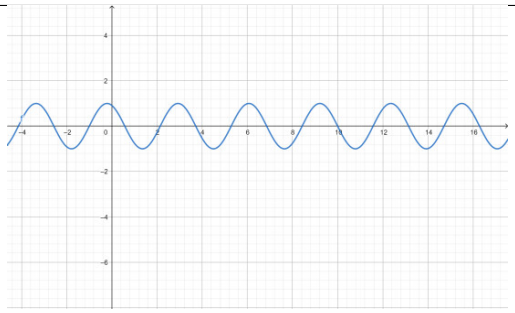
Function	$3\cos(x) - 3$	$3\sin(x)$	$\sin(2x + 2)$	$3\sin(x) + 1$
Graph				



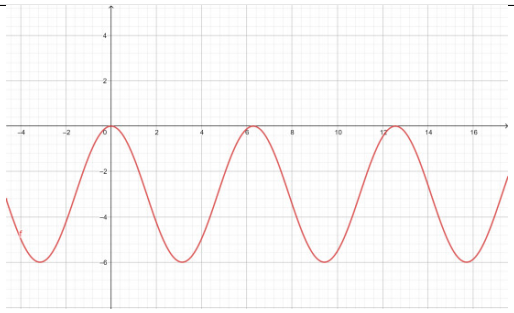
A



B



C



D

Exercise 61 — June 2022 Calc.: ✓

The depth of water at a landing jetty in a small harbour on the North Sea varies according to time due to the tide. There are two tides every day at this harbour.
The depth was measured at 3-hour intervals on 15th June and the following figures were recorded.

Time	00:00	03:00	06:00	09:00	12:00
Depth (m)	3.6	5.2	3.6	2.0	3.6

The depth of water can be modelled by a sine function.

1. **Show that** the function 6

$$h(t) = 1.6 \cdot \sin(0.5236 \cdot t) + 3.6$$

can be used to model the depth of water h (metres), at time t (hours), **explaining** how each of the three constants can be found from the data in the table.

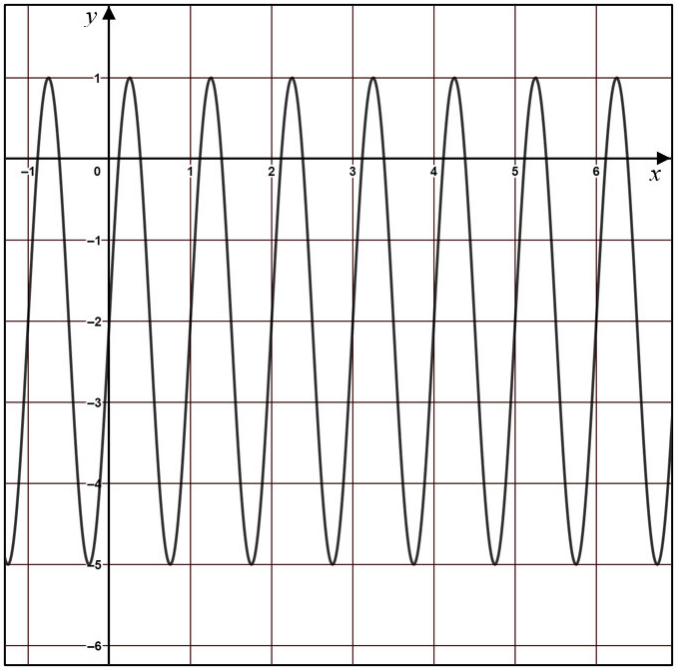
A large ferry from a nearby island requires a minimum depth of 4 m to be able to dock at the jetty.

2. **Show that** the earliest time that the ferry can dock at the jetty on 15th June is 00:29 (rounded to the nearest minute). 3

3. **Find** the latest time before midday when the ferry can dock at the jetty. 3

Exercise 62 — June 2022 Calc.: ✗

The diagram shows the graphical representation of a sine function, f. 7



Determine the Amplitude (a), the Period (p), the Horizontal Translation (c) and the mean value (d) of the function f.

Use these values to **deduce** the equation of the function, $f(x)$.

Exercise 63 — June 2022 Calc.: ✗

For each of the situations A to E described below, state whether the model involves: 10

1. (a) Growth
   (b) Decay
   (c) Neither

and whether the model is:

2. (a) Linear
   (b) Exponential
   (c) Quadratic
   (d) Sinusoidal

A: A population of 100 mice increases by 20% each week under favourable conditions

B: A tree which is 1.2 m tall when planted grows 30 cm each month during the growing season

C: The height, h, of a stone, t seconds after being dropped from the top of a tower is modelled by the function

$$h(t) = 130 - 5t^2$$

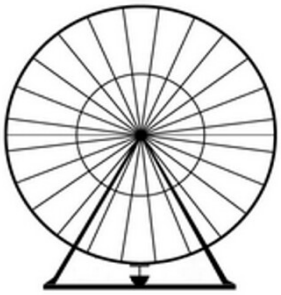
D: The number of daylight hours in Blankenloch varies periodically each year between 16 hrs 12 mins and 8 hrs 13 mins

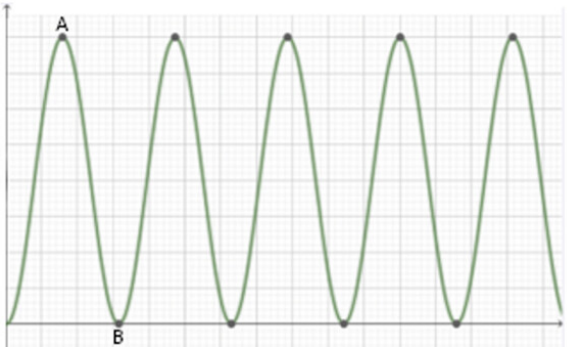
E: The temperature, T, of a liquid, t minutes after being placed in a refrigerator, is given by the function

$$T(t) = 98 \cdot 2^{-\frac{t}{50}}$$

Exercise 64 — June 2022 Calc.: ✗

The graph below shows the height above the ground of a cabin on a Ferris wheel as a function of time.
The Ferris wheel takes 5 minutes to complete one full rotation.
The cabin follows a circular path between the heights of 0 m and 65 m above the ground.





1. **Determine** the coordinates of points A and B on the graph above. 2
2. **Explain** how the graph would change if the Ferris wheel were to take 10 minutes to complete a circuit. 2
3. **Describe** any limitations of this model when applied to the practical situation. 2

Exercise 65 — June 2017 Calc.: ✓

Make sure that calculator is set to radians for this question.
The depth of water at the end of a pier can be estimated by the function

$$d(t) = 5.6 \sin\left(\frac{\pi}{6}t\right) + 14.9$$

Where $d(t)$ is the depth of the water in metres and t is the number of hours after midnight.
Use your calculator to help you to draw a sketch of the graph of this function and to find the following:

1. What is the period of this function? 2
2. Estimate the depth of the water at midnight. 1
3. Estimate the depth of the water at 8am. 1
4. At what time will the water reach its highest point after midday? 2

7 Chapter 7 — Binomial law

Exercise 66 — June 2022 Calc.: ✗

We throw a coin three times in a row to see how many heads or tails we get. Consider the following events:
A : « We get at least two heads ».
B : « We get tails less than three times ».
C : « We get exactly three heads or exactly three tails ».
Verify if the events are independent of each other :

1. Are A and B independent events? 3
2. Are A and C independent events? 3
3. Are B and C independent events? 3

Exercise 67 — June 2022 Calc.: ✓

A fair coin is tossed three times in a row and the results obtained are noted.
For example, "Heads, Heads, Tails" is an outcome that may be noted HHT.

1. **Determine** the probability of getting Heads at least twice. 3

For each toss, 20 points are awarded for Heads and 10 points for Tails.
Let X represent the sum of the points obtained after the three tosses.

2. **Calculate** $P(X = 40)$. 3
3. **Copy** and **complete** the probability distribution table for X shown below. 4

x	30			60
$P(X = x)$	$\frac{1}{8}$			$\frac{1}{8}$

4. **Calculate** the expected value of X and **interpret** this result. 4

Exercise 68 — June 2022 Calc.: ✓

(Give your answers to this question accurate to 4 decimal places where appropriate)
Many squirrels live in the forest around the ESK in Waldstadt.
When a squirrel leaves the forest to go to the trees inside the school grounds, the probability of it being seen by a student is 1/3.
One morning, 10 squirrels decide to go to the trees inside the school grounds.
Let X represent the number of squirrels which are seen by a student.

1. **Calculate** the probability that exactly 7 squirrels will manage to get to the trees in the school grounds without being seen by a student. 4
2. **Calculate** the probability that less than two squirrels will be seen by a student. 4
3. **Calculate** $E(X)$. **Interpret** this result. 4
4. **Calculate** the standard deviation of X. 3

Exercise 69 — June 2022 Calc.: ✗

In a family of 4 people (two parents and their two children), each has a smartphone of the same make and model.
The probability that this "basic" model will fail during the year is 20 %.
Calculate the probability that exactly two of the members of this family will have their smartphone fail during the year. 6

Exercise 70 — June 2022 Calc.: ✗

A tetrahedral dice is labelled with four numbers: 1, 2, 3 and 4.
The dice is thrown three times.
Let X represent the number of times a 1 is obtained.
Determine the probability distribution of the variable X and calculate its expected value. 6

Exercise 71 — June 2022 Calc.: ✓

At a concert there are 135 seats. The organizers of the concert know from experience that only 96% of the people who have bought a ticket will come to the concert. They therefore decide to sell more tickets than there are seats.

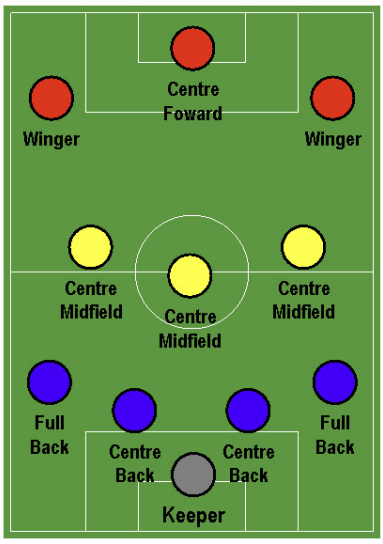
1. Explain why the number of people coming to the concert is a Bernoulli process (binomial experiment). 2
2. The organizers of the concert sell 137 tickets. Calculate the probability of "overbooking". In other words, calculate the probability that more than 135 people will come to the concert. 3

Exercise 72 — June 2022 Calc.: ✓

The coach of a football team selected 24 players for a tournament. He picked 8 defenders, 7 midfielders, 5 strikers and 4 goalkeepers.

1. How many different teams can the coach put together if he chooses a line-up consisting of 1 goalkeeper, 4 defenders, 3 midfielders and 3 strikers? 4

2. The coach has chosen 4 defenders in his line-up: Virgil, Sergio, Ruben and Trent. The trainer can line up these defenders in the places indicated by the blue dots. How many lineups are possible with these 4 defenders? 3



3. For a press conference, a group of 4 players is randomly chosen from all 24 players. Calculate the probability that this group consists of 1 defender, 1 midfielder, 1 striker and 1 goalkeeper. 3

4. The probability that Christiano scores a penalty is 85%. Christiano kicks 5 penalties.

   (a) Calculate the probability that Christiano scores 5 times. 2

   (b) Calculate the probability that Christiano scores 3 out of 5 attempts. 2

   (c) Calculate the probability that Christiano scores 4 times at most. 2

Exercise 73 — June 2019 Calc.: ✓

On a certain remote island, 25 per cent of the population have the gene Hs which is known to protect against malaria. A random sample of 32 people have their blood tested to see if they have this gene.

1. Show that the sample will satisfy the conditions for a Bernoulli trial. 2

2. Determine the probability that the number that are found to have Hs is:

   (a) Exactly 5 1

   (b) Fewer than 10 1

   (c) At least 6 but at most 12 2

   (d) More than the mean value for the distribution. 2

3. Will the conditions for a Bernoulli trial always be true for the population of the island? Write one or two sentences to justify your answer. 2

Exercise 74 — June 2019 Calc.: ✓

In New Portland there are only two car manufactures, Homba and Tayita. 58% of the cars are produced by Homba and the rest is produced by Tayita. No cars by other manufacturers exist.
7% of the cars produced by Homba do not meet the rigorous emission standards set by New Portland's environmental agency, whereas 87% of the cars produced by Tayita do meet these standards.

1. Draw a tree diagram illustrating this situation. All notation must be defined and all branches of the tree must be clearly labelled.	5
2. What is the probability that a car meets the emission standards given that it was produced by Homba?	2
3. What is the probability that a car was produced by Tayita **and** meets the emission standards?	2
4. If a car is selected at random, what is the probability that it meets the emission standards?	3
5. Given that a car meets the emission standards, what is the probability it was produced by Tayita?	3

A study shows that the probability of a car meeting the emission standards is equal to 90%
Ten cars are randomly selected.

6. Write the formula that allows you to calculate the probability that k cars out of the ten chosen meet the emission standards. **The use of the formula must be carefully justified.**	4
7. What is the probability that all cars chosen will meet the emission standards?	2
8. What is the probability that 8 cars meet the emission standards?	2
9. What is the probability that at least 8 cars meet the emission standards?	2

Exercise 75 — June 2017 Calc.: ✓

In an examination hall, it is know that 12% of desks are wobbly.
In a row of 14 desks what is the probability that :

1. There were 0 wobbly desks.	2
2. All the desks were wobbly.	2
3. Less than 4 desks were wobbly.	2