

MATHEMATICS 3 PERIODS PART B

DATE: 12th June 2023, morning

DURATION OF THE EXAMINATION:

2 hours (120 minutes)

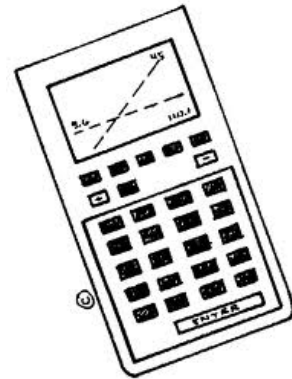
AUTHORISED MATERIAL:

Examination with technological tool:

Approved calculator

Pencil for the graphs

Formelsammlung / Formula booklet / Recueil de formules



SPECIFIC INSTRUCTIONS:

- Use a different page for each question.
- Answers must be supported by explanations.
- Answers must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.

- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.

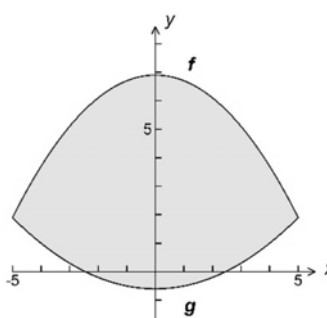
- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.

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PART B		
QUESTION B1	Page 1/3	Marks
<p>Part 1</p> <p>Mary runs a farm.</p> <p>The milk production on the farm can be modelled by the function f given by</p> $f(x) = -0.0028x^2 + 0.57x, \quad 50 \leq x \leq 90,$ <p>where x is the number of cows on the farm and $f(x)$ represents the average daily milk production, measured in hL (1 hL = 1 hectolitre = 100 litres).</p> <p>a) Calculate the average daily milk production of 70 cows.</p> <p>b) Determine how many cows Mary needs to maintain a daily average milk production of 25 hL or more.</p> <p>c) Can the model be extended to 205 cows? Justify your answer.</p>		<p>2 marks</p> <p>3 marks</p> <p>2 marks</p>
<p>Part 2</p> <p>d) The daily summer milk production per cow is normally distributed with mean $\mu = 48$ litres and standard deviation $\sigma = 16$ litres. Calculate the probability that a randomly chosen cow will produce more than 40 litres of milk on a summer's day. Give your answer correct to three decimal places.</p> <p>e) We assume that the probability that a randomly chosen cow will produce more than 40 litres of milk per day is equal to 0.69. Currently Mary has 80 cows. Calculate the probability that less than 60 of these cows produce more than 40 litres of milk per day.</p>		<p>2 marks</p> <p>2 marks</p>

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PART B										
QUESTION B1								Page 2/3	Marks	
<p>Part 3</p> <p>The table below shows the annual rainfall (measured in cm) on the farm over the last 10 years.</p>										
Year	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
$x =$ Years after 2013	0	1	2	3	4	5	6	7	8	9
$y =$ Rainfall (cm)	123	125	117	115	120	113	110	100	108	105
<p>f) Draw a scatter diagram to represent the data from the table and by interpreting this diagram, describe the correlation.</p> <p>g) Determine an equation in the form $y = m \cdot x + b$ of the linear regression of y on x using the data from the table. Draw the regression line on the same diagram.</p> <p>h) Explain why a linear regression model might not be appropriate for this data over many years.</p>								4 marks	4 marks	2 marks

PART B		
QUESTION B1	Page 3/3	Marks
<p>Part 4</p> <p>There is a pond on the farm, a diagram of which you will find below (1 unit = 1 metre):</p>  <p>The boundaries of this pond are the graphs of the functions f and g defined by</p> <p>$f(x) = -0.2x^2 + 6.9$, $-5 \leq x \leq 5$ for the upper boundary and</p> <p>$g(x) = 0.1x^2 - 0.6$, $-5 \leq x \leq 5$ for the lower boundary.</p> <p>i) Calculate the area of this pond.</p>		<p>4 marks</p>

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PART B		
QUESTION B2	Page 1/2	Marks
<p>Part 1</p> <p>a) In August 2021 the trips in Helsinki's bike sharing system had a mean distance of 2.25 km and a standard deviation of 16.04 km.</p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> <p>Explain what could have caused such a large standard deviation.</p> </div> <div style="flex: 1; text-align: center;">  <p><i>Public bikes in Helsinki</i></p> </div> <div style="flex: 0.2; text-align: right; vertical-align: middle;"> <p>2 marks</p> </div> </div> <p>b) Over a certain period, the mean duration of the trips was $\mu = 645$ seconds and the standard deviation was $\sigma = 271$ seconds. Assume that the trip duration is normally distributed.</p> <p>Calculate the probability that a trip took longer than 12 minutes.</p> <div style="text-align: right; vertical-align: middle;"> <p>3 marks</p> </div>		
<p>Part 2</p> <p>A survey covering 2009-2019 has shown that the sale of e-bikes in the European Union can be modelled by the function N given by</p> $N(t) = 0.0756 \cdot e^{0.163t+2.03} ,$ <p>where t is the number of years after 2009 and $N(t)$ is the number of e-bikes sold, in millions.</p> <p>c) Rewrite the formula for $N(t)$ in the form $N(t) = K \cdot A^t$.</p> <div style="text-align: right;"> <p>2 marks</p> </div> <p>d) According to this model, determine the yearly percentage increase in the sale of e-bikes.</p> <div style="text-align: right;"> <p>2 marks</p> </div> <p>e) Since 2009, the total number of all bikes sold (including e-bikes) in Europe has been approximately constant at 20 million bikes per year.</p> <p>Estimate the year in which the number of e-bikes sold will be more than half of all bikes sold.</p> <div style="text-align: right;"> <p>3 marks</p> </div>		

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PART B		
QUESTION B2	Page 2/2	Marks
<p>Part 3</p> <p>The height $h(t)$ in centimetres (cm) of a bicycle pedal above the ground at time t seconds is defined by $h(t) = a \cdot \sin(b \cdot t) + d$.</p> <p>f) The maximum height of the pedal is 49 cm and the minimum height is 9 cm.</p> <p style="padding-left: 40px;">Determine a and d.</p> <p style="text-align: right;">3 marks</p> <p>g) The time taken to complete a full rotation of the pedal is 1.5 seconds.</p> <p style="padding-left: 40px;">Calculate b.</p> <p style="padding-left: 40px;">Explain what information b gives about the rotation of the pedal.</p> <p style="text-align: right;">3 marks</p>		
<p>Part 4</p> <p>On a website (Euro-Velo) for long-distance cycle-routes in Europe, the Rhine Route has been the most visited route.</p> <p>In 2020, 142 124 of the 1 644 417 visitors to the website visited the Rhine Route.</p> <p>In 2021, in a random sample of 2000 visitors to the website, 156 visited the Rhine Route.</p> <p>The Euro-Velo organisation is wondering whether the proportion of people visiting the Rhine Route has decreased from 2020 to 2021. Hence, they are performing a hypothesis test at a 5 % significance level.</p> <p>p denotes the proportion of all visitors to the website visiting the Rhine Route in 2021.</p> <p>h) Verify that the null hypothesis for this test is $H_0 : p = 0.086$.</p> <p style="text-align: right;">2 marks</p> <p>i) Determine whether the test is left or right sided. Justify your answer.</p> <p style="text-align: right;">2 marks</p> <p>j) Calculate the probability that the number of visitors to the Rhine Route from a random sample of 2000 visitors to the website is less than or equal to 156, assuming that H_0 is true.</p> <p style="text-align: right;">3 marks</p> <p>Decide whether H_0 can be rejected. Justify your conclusion.</p>		