MARKING SCHEME

MATHEMATICS 3 PERIODS
PART B

DATE: 12th June 2023, morning

DURATION OF THE EXAMINATION:

2 hours (120 minutes)

AUTHORISED MATERIAL:

Examination with technological tool:

Approved calculator

Pencil for the graphs

Formelsammlung / Formula booklet / Recueil de formules

SPECIFIC INSTRUCTIONS:

- Use a different page for each question.
- Answers must be supported by explanations.
- Answers must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.

PART B

QUESTION B1 — Page 1/5 — Marks

Part 1

Mary runs a farm.

The milk production on the farm can be modelled by the function f given by

$$f(x) = -0.0028x^2 + 0.57x, \quad 50 \le x \le 90,$$

where x is the number of cows on the farm and $f(x)$ represents the average daily milk production, measured in hL (1 hL = 1 hectolitre = 100 litres).

a) **Calculate** the average daily milk production of 70 cows. — 2 marks

The calculator gives

$$f(70) = 26.18$$

The average daily milk production of 70 cows is 26.18 hectolitres (2618 litres).

Writing $f(70)$: 1 mark
Calculating and concluding: 1 mark

b) **Determine** how many cows Mary needs to maintain a daily average milk production of 25 hL or more. — 3 marks

Mathematical representation of the situation:

$$f(x) \ge 25 \quad \text{with} \quad 50 \le x \le 90.$$

The calculator gives

$$63.95 \le x \le 90.$$

At least 64 cows are required to maintain a daily average milk production of 25 hL or more.

Translating mathematically : 1 mark
Determining the interval: 1 mark
Concluding: 1 mark

c) Can the model be extended to 205 cows? **Justify** your answer. — 2 marks

The calculator gives

$$f(205) = -0.82.$$

Using this model, we would find a negative daily average milk production for 205 cows. This model does not apply to a large number of cows.

Calculating $f(205)$: 1 mark
Justifying: 1 mark

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QUESTION B1	Page 2/5	Marks

Part 2

d) The daily summer milk production per cow is normally distributed with mean $\mu = 48$ litres and standard deviation $\sigma = 16$ litres.

Calculate the probability that a randomly chosen cow will produce more than 40 litres of milk on a summer's day. Give your answer correct to 3 decimal places.

2 marks

The random variable X denotes the daily summer milk production per cow.
X follows a normal distribution with mean $\mu = 48$ litres and standard deviation $\sigma = 16$ litres.
The calculator gives

$$P(X > 40) \approx 0.691 .$$

The probability that a randomly chosen cow will produce more than 40 litres of milk on a summer's day is approximately 0.691.

Writing $P(X > 40)$: 1 mark
Calculating and concluding: 1 mark

e) We suppose that the probability that a randomly chosen cow will produce more than 40 litres of milk per day is equal to 0.69.
Currently Mary has 80 cows.

Calculate the probability that less than 60 of these cows produce more than 40 litres of milk per day.

2 marks

The random variable Y denotes the number of cows that produce more than 40 litres of milk per day. Y is binomially distributed with parameters n=80 and p=0.69.

Using the calculator, we find

$$P(Y < 60) = P(Y \leq 59) \approx 0.851.$$

The probability that less than 60 of these cows produce more than 40 litres of milk per day is approximately 0.851, or 85.1%.

Writing $P(Y \leq 59)$: 1 mark
Calculating and concluding: 1 mark

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QUESTION B1	Page 3/5	Marks

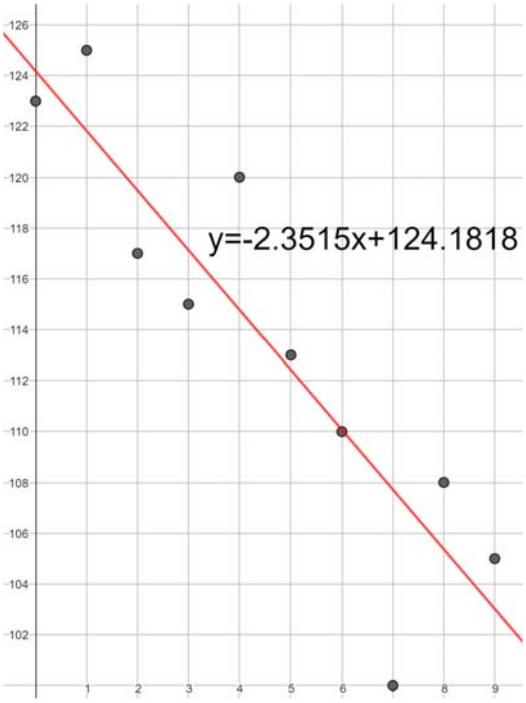
Part 3

The table below shows the annual rainfall (measured in cm) on the farm over the last 10 years.

Year	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
x = Years after 2013	0	1	2	3	4	5	6	7	8	9
y = Rainfall (cm)	123	125	117	115	120	113	110	100	108	105

f) **Draw** a scatter diagram to represent the data from the table and by interpreting this diagram, **describe** the correlation. 4 marks

The figure below belongs to both f) and g).



When x-values increase, the y-values decrease. Over the years, the average annual rainfall in cm on the farm has decreased. There is a rather strong negative correlation.

Drawing a scatter diagram: 2 marks
Describing the correlation: 2 marks

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g) **Determine** an equation in the form $y = m \cdot x + b$ of the linear regression of y on x using the data from the table. **Draw** the regression line on the same diagram. Using the calculator, we find the following equation of the linear regression: $y = -2.3515x + 124.1818$. For the drawing on the diagram, see above. Results like $y = -2.35x + 124.18$ or $y = -2.35x + 124$ are admissible and even more appropriate. (Do not penalise students for giving too many decimal numbers.)		4 marks
Determining an equation of the linear regression line: 2 marks Drawing the regression line on the diagram: 2 marks		
h) **Explain** why a linear regression model might not be appropriate for this data over many years. There are several similar ways of answering. For example: • After a long time, the rainfall would become zero or even negative. Or: • In the year 2066, we would have a negative quantity of rainfall: $x = 53 \Rightarrow y = -0.4477$.		2 marks
Explaining: 2 marks		

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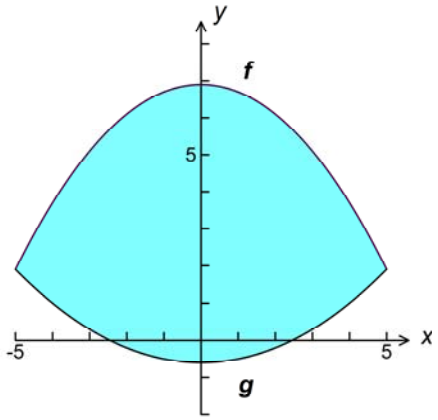
QUESTION B1	**Page 5/5**	**Marks**

Part 4

There is a pond on the farm, a diagram of which you will find below

(unit = 1 metre):



The boundaries of this pond are the graphs of the functions *f* and *g* defined by

$f(x) = -0.2x^2 + 6.9$, $-5 \le x \le 5$ for the upper boundary and

$g(x) = 0.1x^2 - 0.6$, $-5 \le x \le 5$ for the lower boundary.

i) **Calculate** the area of this pond. — 4 marks

The area A (in m^2) of this pond is given by

$A = \int_{-5}^{5} \left[\left(-0.2x^2 + 6.9\right) - \left(0.1x^2 - 0.6\right) \right] dx$. (See formula booklet)

The calculator gives A=50.

The area of the pond is 50 m^2.

Writing the correct formula:1 mark
Calculating the integral: 2 marks
Concluding: 1 mark

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QUESTION B2	Page 1/5	Marks

Part 1

a) In August 2021 the trips in Helsinki's bike sharing system had a mean distance of 2.25 km and a standard deviation of 16.04 km.

Explain what could have caused such a large standard deviation.

Public bikes in Helsinki

The standard deviation gives us information about the spread around the mean of the values. The larger the standard deviation, the more dispersed the values are around the mean. In this example the standard deviation is very large, which means that at least some of the trips were very long.

2 marks

Explaining: 2 marks

b) Over a certain period, the mean duration of the trips was $\mu = 645$ seconds and the standard deviation was $\sigma = 271$ seconds.

Assume that the duration of a trip is normally distributed.

Calculate the probability that a trip took longer than 12 minutes.

X denotes the duration of a trip. X is normally distributed with mean $\mu = 645$ seconds and standard deviation $\sigma = 271$ seconds.
12 min = 720 s.
Using the calculator we get:
$P(\text{Trip took more than 720 seconds}) = P(X > 720) \approx 0.391$.
The probability that a trip took more than 12 minutes is approximately 0.391 or 39.1%.

3 marks

Writing $P(X > 720)$: 1 mark
Calculating and concluding: 2 marks

PART B

QUESTION B2	Page 2/5	Marks

Part 2

A survey covering 2009-2019 has shown that the sale of e-bikes in the European Union can be modelled by the function N given by

$N(t) = 0.0756 \cdot e^{0.163t+2.03}$,

where t is the number of years after 2009 and $N(t)$ is the number of e-bikes sold, in millions.

c) **Rewrite** the formula for $N(t)$ in the form $N(t) = K \cdot A^t$. — 2 marks

$N(t) = 0.0756 \cdot e^{0.163t+2.03} = \left(0.0756 \cdot e^{2.03}\right) \cdot e^{0.163t}$

$\approx 0.576 \cdot (e^{0.163})^t$

$\approx 0.576 \cdot 1.177^t$ } (using the calculator).

Isolating $e^{2.03}$: 1 mark
Giving the required form: 1 mark

d) According to this model, **determine** the yearly percentage increase in the sale of e-bikes. — 2 marks

$N(t) = 0.576 \cdot (1+0.177)^t$.

Hence, the yearly percentage increase in the sale of e-bikes is 17.7%.

Or: $\frac{N(t+1)}{N(t)} = \frac{0.0756 \cdot e^{0.163(t+1)+2.03}}{0.0756 \cdot e^{0.163t+2.03}} = e^{0.163} \approx 1.177.$ … (using the calculator)

Determining the yearly percentage increase: 2 marks

e) Since 2009, the total number of all bikes sold (including e-bikes) in Europe has been approximately constant at 20 million bikes per year. — 3 marks

Estimate the year in which the number of e-bikes sold will be more than half of all bikes sold.

50% of 20 million bikes is 10 million bikes.
Solve $N(t) = 10$. The calculator gives $t \approx 17.5$.
We check the years 2026 $(t = 17)$ and 2027 $(t = 18)$:
$N(17) \approx 9.2$
$N(18) \approx 10.8$.
Thus, from the year 2027, more than 10 million electric bicycles will be sold, which represents a market share of more than 50%.

Calculating 50% of 20 million: 0.5 marks
Writing the equation $N(t) = 10$: 0.5 marks
Solving the equation: 1 mark
Estimating the year: 1 mark

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QUESTION B2	Page 3/5	Marks

Part 3

The height $h(t)$ in centimetres (cm) of a bicycle pedal above the ground at time t seconds is defined by $h(t) = a \cdot \sin(b \cdot t) + d$.

f) The maximum height of the pedal is 49 cm and the minimum height is 9 cm.
Determine a and d. — 3 marks

The amplitude a is half the distance between the maximum and minimum pedal heights (the length of the crank) : $a = \frac{49-9}{2} = 20$.

d is the vertical shift. $d = 9 + 20 = 29$.

Hence, the amplitude is 20 centimetres and the vertical shift is 29 centimetres.

Determining a : 1 mark
Determining d : 1 mark
Concluding with units: 1 mark

g) The time taken to complete a full rotation of the pedal is 1.5 seconds. — 3 marks

Calculate b.

Explain what information b gives about the rotation of the pedal.

$$p = \frac{2\pi}{b} \Leftrightarrow 1.5 = \frac{2\pi}{b} \Leftrightarrow b = \frac{2\pi}{1.5} \Leftrightarrow b = \frac{4\pi}{3}.$$

The pedal moves $\frac{4\pi}{3}$ radians per second or 240° per second.

Calculating b : 2 marks
Explaining: 1 mark

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QUESTION B2	**Page 4/5**	**Marks**

Part 4

On a website (Euro-Velo) for long-distance cycle-routes in Europe, the Rhine Route has been the most visited route.

In 2020, 142 124 of the 1 644 417 visitors to the website visited the Rhine Route.

In 2021, in a random sample of 2000 visitors to the website, 156 visited the Rhine Route.

The Euro-Velo organisation is wondering whether the proportion of people visiting the Rhine Route has decreased from 2020 to 2021. Hence, they are performing a hypothesis test at a 5 % significance level.

p denotes the proportion of all visitors to the website visiting the Rhine Route in 2021.

h) **Verify** that the null hypothesis for this test is $H_0 : p = 0.086$. — 2 marks

The null hypothesis H_0 is the hypothesis that there is no difference between the proportions of people visiting the Rhine Route in 2020 and in 2021.

In 2020 the proportion of visitors to the website that visited the Rhine Route is $\frac{142\ 124}{1\ 644\ 417} = 0.08643$.

Hence: $H_0 : p \simeq 0.086$

Explaining the meaning of the null hypothesis: 1 mark
Determining H_0: 1 mark

i) **Determine** whether the test is left or right sided. **Justify** your answer. — 2 marks

The organisation wants to know whether the proportion of people visiting the Rhine Route has decreased from 2020 to 2021. Thus, the test is left sided.

Arguing: 1 mark
Concluding: 1 mark

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QUESTION B2	**Page 5/5**	**Marks**
j) **Calculate** the probability that the number of visitors to the Rhine Route from a random sample of 2000 visitors to the website is less than or equal to 156, assuming that H_0 is true. **Decide** whether H_0 can be rejected. **Justify** your conclusion. X denotes the number of visitors to the Rhine Route among 2000 visitors to the website. X follows a binomial distribution with parameters $n = 2000$ and $p = 0.086$. The calculator gives $P(X \leq 156) = 0.107$ or 10.7 %, which is much more than the 5 % significance level. Hence the sample proportion has not decreased significantly from what we have hypothesized in the Null-hypothesis. There is a decrease, but it is not a significant decrease. Conclusion: H_0 cannot be rejected.		3 marks
Choosing the binomial distribution with appropriate parameters: 1 mark Calculating $P(X \leq 156)$: 1 mark Concluding: 1 mark		