Exercise 1 Calc. : ✓

Part 1

Mary runs a farm.
The milk production on the farm can be modelled by the function f given by

$$f(x) = -0.0028x^2 + 0,57x, \qquad 50 \leq x \leq 90,$$

where x is the number of cows on the farm and $f(x)$ represents the average daily milk production, measured in hL (1 hL = 1 hectolitre = 100 litres).

a) **Calculate** the average daily milk production of 70 cows. — 2 marks

b) **Determine** how many cows Mary needs to maintain a daily average milk production of 25 hL or more. — 3 marks

c) Can the model be extended to 205 cows? **Justify** your answer. — 2 marks

Part 2

d) The daily summer milk production per cow is normally distributed with mean $\mu = 48$ litres and standard deviation $\sigma = 16$ litres.

Calculate the probability that a randomly chosen cow will produce more than 40 litres of milk on a summer's day. Give your answer correct to three decimal places. — 2 marks

e) We assume that the probability that a randomly chosen cow will produce more than 40 litres of milk per day is equal to 0.69. Currently Mary has 80 cows.

Calculate the probability that less than 60 of these cows produce more than 40 litres of milk per day. — 2 marks

Part 3

The table below shows the annual rainfall (measured in cm) on the farm over the last 10 years.

Year	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
x = Years after 2013	0	1	2	3	4	5	6	7	8	9
y = Rainfall (cm)	123	125	117	115	120	113	110	100	108	105

f) **Draw** a scatter diagram to represent the data from the table and by interpreting this diagram, **describe** the correlation. — 4 marks

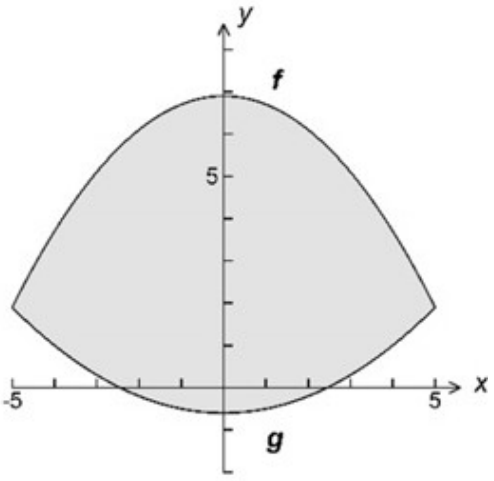
g) **Determine** an equation in the form $y = m \cdot x + b$ of the linear regression of y on x using the data from the table. — 4 marks

Draw the regression line on the same diagram.

h) **Explain** why a linear regression model might not be appropriate for this data over many years. — 2 marks

Part 4

There is a pond on the farm, a diagram of which you will find below (1 unit = 1 metre) :



The boundaries of this pond are the graphs of the functions f and g defined by
$f(x) = -0.2x^2 + 6.9, \quad -5 \leq x \leq 5$ for the upper boundary and
$g(x) = 0.1x^2 - 0.6, \quad -5 \leq x \leq 5$ for the lower boundary.

i) **Calculate** the area of this pond. 4 marks

Exercise 2 Calc. : ✓

Part 1

a) In August 2021 the trips in Helsinki's bike sharing system had a mean distance of 2.25 km and a standard deviation of 16.04 km.

Explain what could have caused such a large standard deviation. — 2 marks

Public bikes in Helsinki

b) Over a certain period, the mean duration of the trips was $\mu = 645$ seconds and the standard deviation was $\sigma = 271$ seconds. Assume that the trip duration is normally distributed.

Calculate the probability that a trip took longer than 12 minutes. — 3 marks

Part 2

A survey covering 2009–2019 has shown that the sale of e-bikes in the European Union can be modelled by the function N given by

$$N(t) = 0.0756 \cdot e^{0.163t+2.03},$$

where t is the number of years after 2009 and $N(t)$ is the number of e-bikes sold, in millions.

c) **Rewrite** the formula for $N(t)$ in the form $N(t) = K \cdot A^t$. — 2 marks

d) According to this model, **determine** the yearly percentage increase in the sale of e-bikes. — 2 marks

e) Since 2009, the total number of all bikes sold (including e-bikes) in Europe has been approximately constant at 20 million bikes per year.

Estimate the year in which the number of e-bikes sold will be more than half of all bikes sold. — 3 marks

Part 3

The height $h(t)$ in centimetres (cm) of a bicycle pedal above the ground at time t seconds is defined by $h(t) = a \cdot \sin(b \cdot t) + d$.

f) The maximum height of the pedal is 49 cm and the minimum height is 9 cm.

Determine a and d. 3 marks

g) The time taken to complete a full rotation of the pedal is 1.5 seconds.

Calculate b. 3 marks

Explain what information b gives about the rotation of the pedal.

Part 4

On a website (Euro-Velo) for long-distance cycle-routes in Europe, the Rhine Route has been the most visited route.
In 2020, 142 124 of the 1 644 417 visitors to the website visited the Rhine Route.
In 2021, in a random sample of 2 000 visitors to the website, 156 visited the Rhine Route.

The Euro-Velo organisation is wondering whether the proportion of people visiting the Rhine Route has decreased from 2020 to 2021. Hence, they are performing a hypothesis test at a 5% significance level.
p denotes the proportion of all visitors to the website visiting the Rhine Route in 2021.

h) **Verify** that the null hypothesis for this test is $H_0 : p = 0.086$. 2 marks

i) **Determine** whether the test is left or right sided. **Justify** your answer. 2 marks

j) **Calculate** the probability that the number of visitors to the Rhine Route from a random sample of 2 000 visitors to the website is less than or equal to 156, assuming that H_0 is true. 3 marks

Decide whether H_0 can be rejected. **Justify** your conclusion.