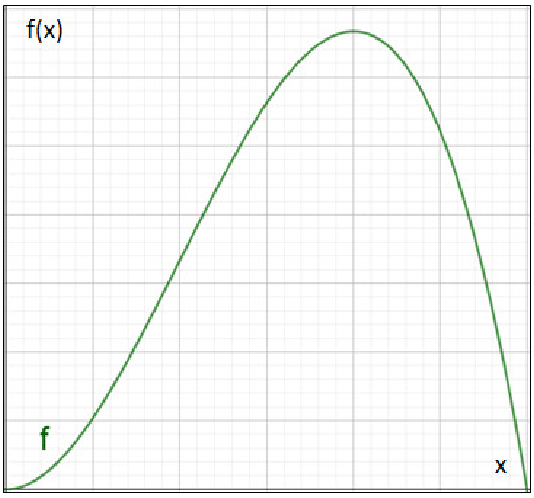
Exercise 1 Calc. : ✗

A little hill on a playground can be modelled by a function f with $f(x) = -\frac{1}{3}x^3 + x^2$, for $x > 0$ where x is the distance in m and $f(x)$ is the height in m. The picture shows the graph of this function f.



Determine the height of this hill. 5 marks

Exercise 2 Calc. : ✗

After some complaints about the lunch in the canteen, the manager claims that at most only 20% of all 2,500 pupils are not satisfied with the lunch. The pupils committee thinks that it is more than 20% of the pupils. So, they ask a group of 40 randomly chosen pupils for their opinion.

1. **Explain**, whether a left or a right sided test should be used to verify this hypothesis. Reason your answer. 2 marks
2. **State** which null hypothesis H_0 could be used for a NHST test and give the alternative hypothesis H_1. 1 mark
3. **Determine** the critical value k with the help of the following table if the significance level is set at 5% and **interpret** this value. 2 marks

k	8	9	10	11	12	13	14	15
$P(X \geq k)$	0.563	0.407	0.268	0.161	0.088	0.043	0.019	0.008

Exercise 3 Calc. : ✗

A small supermarket chain employs 900 people, 10 of them work in the management, but only one of the managers is female. The other 809 women work in the shops.
Show, that it depends on the sex, if you get a position in the management of this company. 5 marks

Exercise 4 Calc. : ✗

A couple needs a negative Covid test to visit friends abroad. It is known that 20% of the tests show a negative result, although the person could be infected (false negative result). The probability of a false positive result is close to zero. It can be assumed that if one of them is infected, that the other one is also infected.
Explain, why this situation is a Bernoulli process and **show**, that the probability of a false negative result drops down to 4% when both get tested. 5 marks

Exercise 5 Calc. : ✗

In the diagram shown below, the number of new Covid-19 cases in Germany is shown over a period of 3 weeks in October 2020. To predict the numbers in the future, two basic types of mathematical models can be combined.

Give the names of these types of models and **reason** your answer. — 5 marks

Predict a date in the future when another maximum will be reached, when the numbers follow the models.

Exercise 6 Calc. : ✗

The number of bacteria in a petri dish is investigated in a laboratory. It turns out, that under certain conditions, the growth can be modelled by the function

$$N(t) = 10\ 000 \cdot e^{\ln(1.03)\cdot t},$$

where $N(t)$ is the number of bacteria after t days.

1. **Give** the number of bacteria at the beginning and the growth rate [per day] in percent. — 2 marks
2. **Calculate** the number of bacteria after the first day. — 2 marks
3. **Explain**, why this model cannot be used on a very large time scale. — 1 mark

Exercise 7 Calc. : ✗

Indicate if the statement is true or false and **reason** your answer. Note that the points are only given if answer and reason are correct.

1. If the temperature $T(x)$ is constantly increasing, then $T'(x) > 0$. — 1 mark
2. All periodic models can be modelled by a sine function. — 1 mark
3. There are 9 different possibilities for 3 pupils to stand next to each other. — 1 mark
4. When some die [well balanced with 6 faces numbered from 1 to 6] is rolled once, the expected value is 3.5. — 1 mark
5. If 10 people are chosen out of a very large group, the number of [chosen] females can be modelled by a binomial distribution, although a person cannot be chosen more than once. — 1 mark

Exercise 8 Calc. : ✗

The daylength $L(t)$ in hours on a certain location was recorded over one year. It can be modelled by the function $$L(t) = 4 \cdot \sin\left(\frac{2\pi}{365}t\right) + 12,$$ where t is the time in days. **Interpret** the outcome of $\int_0^{365} L(t)\,\mathrm{d}t$ and **explain**, why the result is equal to $12 \cdot 365 = 4\,380$.	5 marks

Exercise 9 Calc. : ✗

1. **Interpret** what is meant by expected value of a random variable.	2 marks
2. X is a random variable following a normal distribution with expected value μ and standard deviation σ. **Give** a probability taking into account these two characteristic values μ and σ.	1 mark
3. A continuous random variable Y defined over $\mathbb{R}$ is such that $P(a \leq Y \leq b) = \int_a^b f(z)\,\mathrm{d}z$. **Explain** why $\int_{-\infty}^{+\infty} f(z)\,\mathrm{d}z = 1$.	2 marks

Exercise 10 Calc. : ✗

A new machine recognises doping in blood. Let there be the following two events: • P: The test is positive • D: The sportsman was doped After some test runs it was found out, that out of 100 blood samples with doping, the machine recognises it in 90 cases. However, it also gives a false alarm in 5% of the cases, when the sample was clean. It can be assumed, that every 10$^{\text{th}}$ sportsman at a certain event is doped. We want to find out the probability that a sportsman was indeed doped when the test is positive. 1. **Present** all necessary information in the correct mathematical notation. 2. **Use** an appropriate method to determine the probability for a sportsman to be doped when the machine gives an alarm.	5 marks