

Exercise 1

Calc. : ✓

Gabriella is playing with her remote-controlled toy car. The following equation describes the path of the car:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 1 \end{pmatrix} + t \begin{pmatrix} -12 \\ 5 \end{pmatrix}$$

The distance units are metres, and the time is in minutes.

1. Write down the initial position of the car.
2. Calculate the position of the car after 15 seconds.
3. Compute the speed of the car.

1 mark

1 mark

1 mark

Grandma is watching Gabriela from point P(-1, -6)

4. Find the shortest distance from point P to the path of the car.

3 marks

The edge of the cliff is at the point $\left(0, \frac{23}{3}\right)$ and Grandma walks in that direction with velocity vector $\begin{pmatrix} 3 \\ 41 \end{pmatrix}$.

5. After how many minutes will the car reach the edge of the cliff?

2 marks

6. Will Grandma be able to catch the car before it falls down the cliff if she starts moving at the same time as the car? Explain your answer.

4 marks

Exercise 2

Calc. : ✓

1. A contractor must carry out work for a public body. If they do not complete the work on time, they will have to pay a daily penalty: 100 € on the first day, 110 € on the second day, and so on with a daily increase of 10 € a day.

Let u_n be the penalty on the n -th day. Thus, the first term in sequence u is $u_1 = 100$.

- (a) State the nature and characteristics of sequence u .

1 mark

- (b) Explain why $u_n = 90 + 10n$ for all values of integer n .

1.5 marks

- (c) On what day would the daily penalty amount to 220 €?

1 mark

- (d) What total amount of penalty would the contractor have paid after 20 days of delay?

2.5 marks

2. On another construction site, the penalty for delay is 80 € on the first day and then increases by 10% each day. Let v_n be the amount of the penalty on day n in this case.

- (a) Compute the values of the first three terms v_1 , v_2 and v_3 .

1.5 marks

- (b) Explain why $v_n = 80 \cdot 1.10^{n-1}$ for all values of integer n .

1.5 marks

- (c) What is the total amount of penalty the contractor would have paid after 20 days of delay?

2 marks

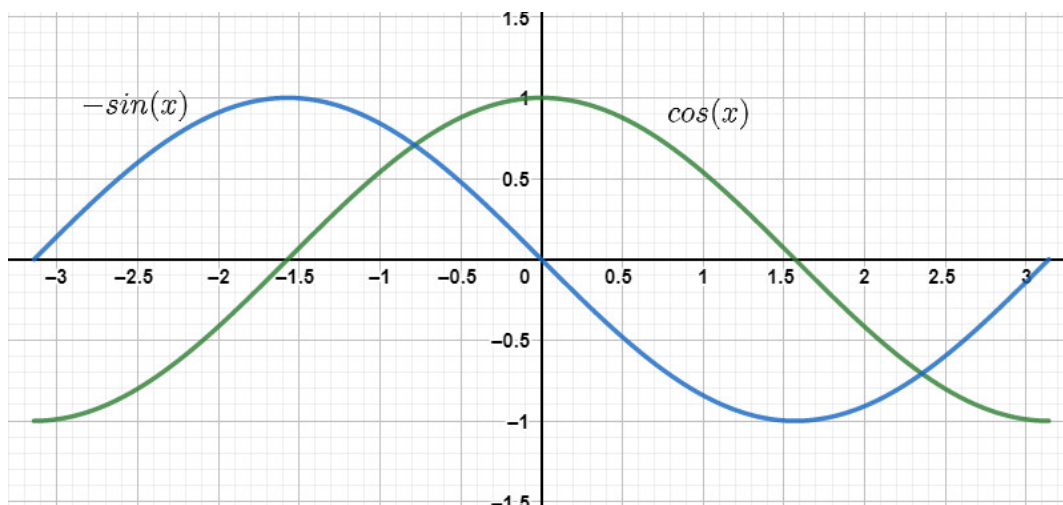
3. From which day onwards does the amount of the daily penalty in this case exceed that of the first case?

3 marks

Exercise 3

Calc. : ✓

1. We consider the functions $x \mapsto \cos x$ and $x \mapsto -\sin x$ on $[-\pi; \pi]$ and their graphic representations below:



Justify that the only solutions of the equation $\cos x + \sin x = 0$ on $[-\pi; \pi]$ are $\frac{-\pi}{4}$ and $\frac{3\pi}{4}$.

3 marks

2. Let f be the function defined on $[-\pi; \pi]$ by: $f(x) = e^x \cdot \sin x$

We note C_f its representative curve in a coordinate system.

- Determine the variations of the function f on $[-\pi; \pi]$, specifying the abscissa, the value and the nature of each extremum.
- Determine an equation of the tangent to the curve C_f at the point of abscissa $\frac{\pi}{2}$.
- On what interval is C_f entirely above each of its tangents? To justify.
- Using two successive integrations by parts, calculate the exact value of the integral:

2 marks

2 marks

2 marks

2 marks

$$\int_{-\pi}^{\pi} f(x) dx.$$

Exercise 4

Calc. : ✓

A company is conducting a study into the relationship between the experience and salary of their staff. The experience and salaries of 12 employees were tabulated.

| | | | | | | | | | | | | |
|---------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Experience x (years) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| Salary y (€) | 4 200 | 4 800 | 4 600 | 5 000 | 5 200 | 5 600 | 5 650 | 5 660 | 5 500 | 6 000 | 5 831 | 6 200 |

1. One of the following correlation coefficients fits these data. Which is it?

1 mark

$$r_1 = 0.95, \quad r_2 = -0.95 \quad \text{or} \quad r_3 = 1?$$

Explain without referring to any computations.

2. Compute the coordinates of the average point for these data, to the nearest integer.

2 marks

3. The equation of regression line with the method of the least squares is $y = a + bx$, where

2 marks

$$b = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad \text{and} \quad a = \bar{y} - b\bar{x}.$$

Use the information given below to compute the values of coefficients a and b . Give answers to 2 decimal places.

| x_i | $x_i - \bar{x}$ | $(x_i - \bar{x})^2$ |
|-------|-----------------|---------------------|
| 0 | -11 | 121 |
| 2 | -9 | 81 |
| 4 | -7 | 49 |
| 6 | -5 | 25 |
| 8 | -3 | 9 |
| 10 | -1 | 1 |
| 12 | 1 | 1 |
| 14 | 3 | 9 |
| 16 | 5 | 25 |
| 18 | 7 | 49 |
| 20 | 9 | 81 |
| 22 | 11 | 121 |

$$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = 45\,009$$

4. Use the linear model $f(x) = 78.7x + 4\,488$ to estimate the salary of an employee with 40 years of experience.

2 marks

The salaries of the employees of this company are normally distributed with mean $\mu = 5\,353$ and standard deviation $\sigma = 553$.

5. Mr. Smith, an employee of this company, is paid 6 459 €. What proportion of the employees of this Company are paid less than Mr. Smith?

1.5 marks

6. Compute the probability that an employee's salary is greater than 7 636 € and comment your answer for question 5.

1.5 marks

In another company, the salaries are normally distributed with standard deviation $s = 620$.

7. Knowing that the probability that an employee's salary is greater than 5 000 € is approximately 0.107, find the mean salary in that company. Write your answer to the nearest whole number.

3 marks