

**Exercise 1**

Calc. : ✓

The functions  $f$  and  $g$  are defined to be

$$\begin{cases} f(x) = x^2 + 2x & x \in \mathbb{R} \\ g(x) = \frac{1}{x+2} & x \in \mathbb{R}, x \neq -2. \end{cases}$$

1. Draw a sketch of the graph of  $f$  labelling clearly the coordinates for the vertex and all points where the graph intersects the coordinate axes. 3 marks

The range for  $g$  is written  $g(x) \in \mathbb{R}, g(x) \neq 0$ .

2. What is the range for  $f$ ? 2 marks
3. Write explicitly an expression for the composite function  $g(f(x))$  and thus evaluate  $g(f(2))$ . 1 mark
4. Solve the equation  $g(f(x)) = \frac{1}{10}$ . 2 marks
5. Is the function  $g(f(x-1))$  an odd function or an even function? 2 marks  
Give a reason for your answer.

**Exercise 2**

Calc. : ✓

You must justify your answers by writing all calculations that are relevant to your solutions.

1. The 4th term of a geometric series is 10 and the 7th term is 80. 5 marks  
Use this information to find the common ratio and the first term for this series and hence find the sum of the first 15 terms.
2. A different series begins by adding the following numbers 5 marks

$$14 + 19 + 24 + \dots$$

How many terms of this series must be added together in order to exceed the sum of the first 15 terms of the geometric series?

**Exercise 3**

Calc. : ✓

The function  $y = 2x^3 - 5x^2 - 4x + 2$  is defined for  $x \in \mathbb{R}$ .

1. Use differentiation to determine the  $(x, y)$  coordinate for any stationary points of the function  $y$ . 2 marks
2. Classify the nature of any stationary points in terms of local maxima or minima. 3 marks
3. Find the range of  $x$  values for which the curve is increasing. 2 marks
4. Find the equation of the tangent line at  $x = 1$ . 3 marks

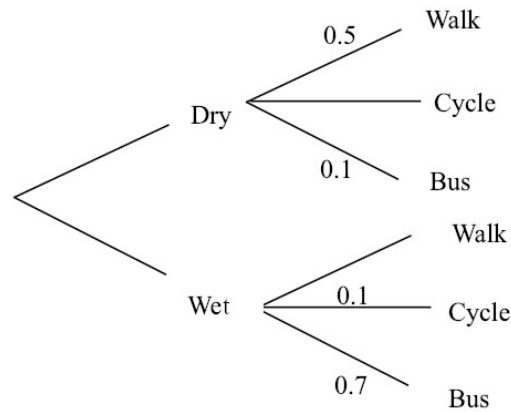
**Exercise 4**

Calc. : ✓

Andy can walk to work, cycle or travel by bus. The choice he makes depends on the weather.

1. Copy and complete the tree diagram to show the probabilities for each of Andy's methods of travel.

2 marks



2. A day is selected at random. Calculate the probability of the following events:

- (a) The weather is wet and Andy travels by bus.  
 (b) Andy walks or cycles.  
 (c) The weather is dry given that Andy walks or cycles.

2 marks

3 marks

3 marks

**Exercise 5**

Calc. : ✓

On a certain remote island, 25 per cent of the population have the gene  $\underline{Hs}$  which is known to protect against malaria.

A random sample of 32 people have their blood tested to see if they have this gene.

1. Show that the sample will satisfy the conditions for a Bernoulli trial.  
 2. Determine the probability that the number that are found to have  $\underline{Hs}$  is:  
 (a) Exactly 5  
 (b) Fewer than 10  
 (c) At least 6 but at most 12  
 (d) More than the mean value for the distribution.

2 marks

1 mark

1 mark

2 marks

2 marks

3. Will the conditions for a Bernoulli trial always be true for the population of the island? Write one or two sentences to justify your answer.

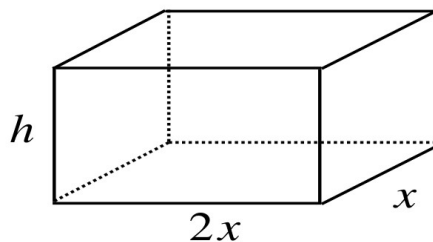
2 marks

**Exercise 6**

Calc. : ✓

The length of the base of a cuboid is twice the width  $x$ , and its height is  $h$  centimetres, as shown in the diagram below.

Its total surface area is  $A$  cm<sup>2</sup> and its volume is  $V$  cm<sup>3</sup>.



1. Show that  $A = 4x^2 + 6xh$ .

2 marks

The manufacturer needs the total surface area  $A = 300$  cm<sup>2</sup>.

2. Find an expression for  $h$ , in terms of  $x$ .

2 marks

3. Show that the volume  $V(x) = 100x - \frac{4}{3}x^3$ .

1 mark

4. Determine the maximum volume possible for the cuboid and determine the value of  $h$  that achieves this.

5 marks

You should explain in your answer how you know that this will be a maximum value.