Exercise 1 Calc. : ✓

For this question you are reminded of the following formulae:

- the volume of the solid of revolution generated by rotating the area under the graph of $y = f(x)$ around the x-axis limited by the lines $x = a$ and $x = b$ is given by:

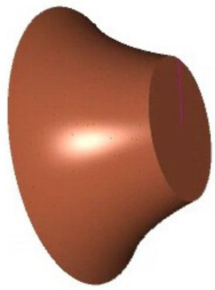
$$V = \int_a^b \pi \left(f(x)\right)^2 \, dx$$

- the length of an arc along the graph of $y = f(x)$ from $x = a$ to $x = b$ is given by:

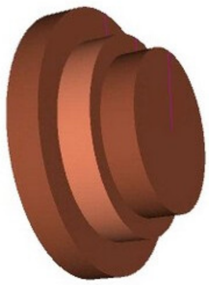
$$s = \int_a^b \sqrt{1 + \left(f'(x)\right)^2} \, dx$$

- the volume of a cylinder with radius r and height h is given by $\pi r^2 h$.

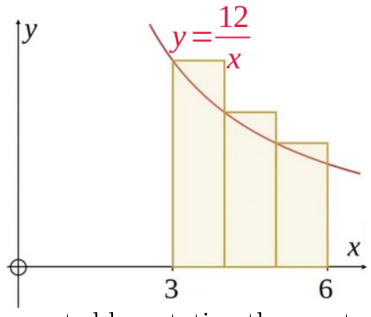
A carpenter uses a machine called a lathe to shape wood so that it always has a circular cross section.
She wishes to produce the final solid shown on the right:



She starts with 3 cylinders glued together as shown.



The original cylinders correspond to the volume of revolution created by rotating the "left rectangles" shown in the graph on the right, where the curve is given by $y = \dfrac{12}{x}$ and the three rectangles are equal in width between between $x = 3$ and $x = 6$.
The unit of length for x and y is 1 cm.



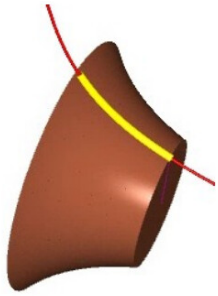
The final smooth solid corresponds to the volume of revolution created by rotating the exact area under the curve around the x-axis, also between $x = 3$ and $x = 6$.

Giving your answers in cm^3 to one decimal place:

a) **Calculate** the total volume of the original cylinders. — 3 marks

b) **Calculate** the volume of wood left in the final smooth solid. — 3 marks

The carpenter will decorate the solid with a pure gold arc that follows the curve $y = \dfrac{12}{x}$ as shown.
The gold line costs 2 € per metre length.



c) **Calculate** the length of the gold arc to 3 decimal places of cm, and hence **estimate** the total cost of the gold to the nearest centime if the carpenter decorates 30 copies of the solid. — 4 marks

The carpenter labels the solid J and then produces nine more similar solid shapes of various sizes labelling them from A to I.
She intended to use the same type of wood for the new solids as she did for J, but after completing the work she realises that she used a denser wood for two of them, and she is not sure which.
She calculates the volume and finds the mass of each of the new solids, recording the results in the table show next page.

	A	B	C	D	E	F	G	H	I	J
Volume (cm^3)	80.5	63.4	90.8	53.4	71.7	105.6	88.8	66.9	99.7	
Mass (g)	36.0	28.5	45.5	24.0	32.5	47.5	44.5	30.0	45.0	

d) Use your calculator to **find** (with coefficients to 2 decimal places) an equation of the line of best fit and the correlation coefficient for the solids A to I. — 3 marks

e) **Identify** which of A to I are made of denser wood. Remove them from the data and then find (again to 2 decimal places) an equation of the line of best fit and the correlation coefficient for the remaining seven solids. — 3 marks

f) J has a mass of 34 g. **Determine** an approximation for the volume, in cm^3 to 1 decimal place, of J from your answer to e) and compare it with your answer to b). **Comment** on the results. — 3 marks

A reminder that density is calculated as mass per unit volume, in this in case grams/cm^3.

g) **Explain** how to use the line of best fit to calculate the density of wood for the eight solids made from the same type of wood. — 3 marks

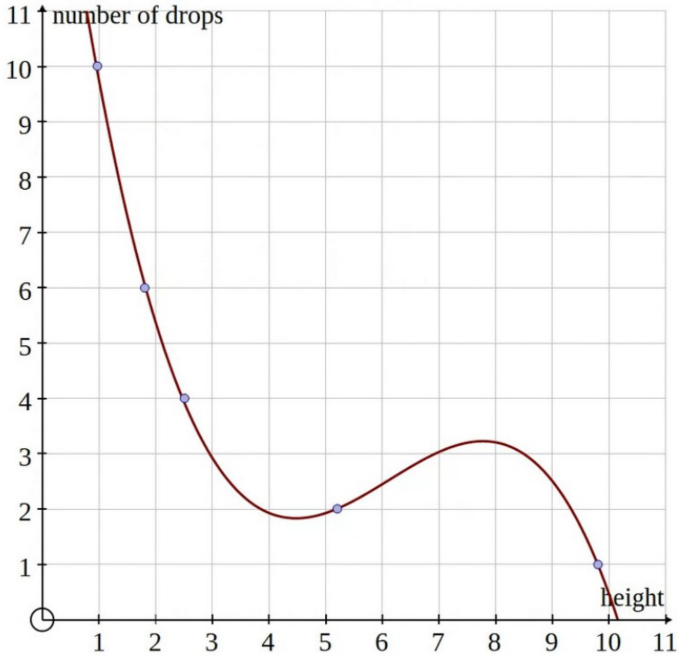
h) **Calculate**, to 1 decimal place in grams/cm^3, the density of the wood used for the other 2 solids. — 3 marks

Exercise 2 Calc. : ✓

Crows can break open walnuts by dropping them from a tree onto a hard surface.
If the walnut does not break the first time the crow flies back to the tree and drops it again.

Observation of crows doing this with similar walnuts gave the following results, and the graph on the right shows the data with a cubic curve that fits the data well.

height in metres, x	number of drops, y
0.96	10
1.8	6
2.5	4
5.2	2
9.8	1



a) **Explain** why the cubic seems a poor model for this situation. 4 marks

b) Using Napierian logarithms **find** the logarithm, to 2 decimal places, of each item of data above and **create** a table on your answer sheet as shown below: 3 marks

$\ln x$	$\ln y$
...	...

c) **Find** the line of best fit for the logarithm data in the form $\ln y = a \ln x + b$, stating the values of a and b to 2 decimal places. 3 marks

d) Using the model from part c), **find** an approximation to the nearest cm for the height from which a crow would have to drop a walnut to break it at the fifth attempt. 4 marks

A crow realises that a walnut it has picked up has gone bad and drops it while flying over a lake. It takes 2 seconds to hit the water.
The vertical velocity of the walnut over time is given by:

- $v_1(t) = -10t + 0.5t^3$ before it hits the water
- $v_2(t) = t - 4$ while in the water until it come back to the surface,

where t is the number of seconds after the walnut is released by the crow and v_1 and v_2 are the velocities in m/s. A negative velocity means that the walnut is falling.

e) **Determine** the height above the water when the crow releases the walnut. 4 marks

f) **Find** the maximum depth of the walnut in the water. 4 marks

g) **Justify** that the walnut is under water for 4 seconds. 3 marks