Exercise 1 Calc. : ✓

Tom and Simon play a board game. Each time Tom manages to move his piece around the board he gets 5 points. Each time Simon manages to move his piece around the board he gets 10% of the previous amount. They both start with 10 points.

1. **Calculate** Tom's total score after moving around the board 20 times. — 2 marks
2. **Write** in terms of n the formula $T(n)$ for Tom's score after n moves around the board. — 2 marks
3. If you know that Simon's score after n moves around the board could be modelled with a geometric sequence, **explain** the use of the formula: — 2 marks

$$S(n) = 11 \cdot 1.1^{n-1}$$

4. Simon and Tom have been around the board the same number of times. Simon's score has just moved ahead of Tom's.

   Find how many times have they been around the board. — 3 marks

Tom challenges Simon to a dice game. Two fair six-sided die are rolled and the sum of scores is noted. For a sum less than 6 Simon receives 10 cents, for a sum between 6 and 9 Simon loses 5 cents, and for the sum bigger or equal 10 Simon receives 30 cents. The winnings are governed by the probability distribution shown below, where the random variable N is the sum of scores.

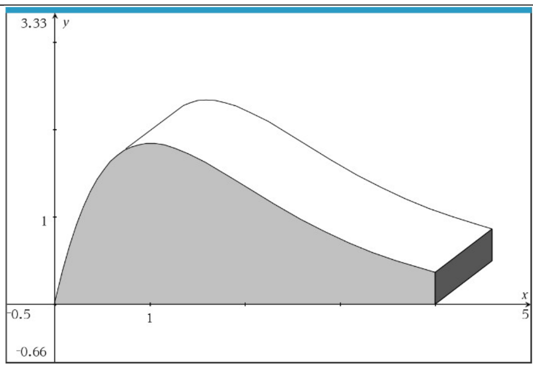
N	$n < 6$	$6 \geq n \geq 9$	$n \geq 10$
Winnings n	10 cents	-5 cents	30 cents
$P(N = n)$	a	$\frac{20}{36}$	b

5. **Show**, that $a = \frac{10}{36}$ and $b = \frac{6}{36}$. — 2 marks
6. **Calculate** the expected value of Simon's winnings in this game and comment if it is worth Simon playing. — 2 marks
7. A game is said to be fair if the expected value is 0. — 2 marks

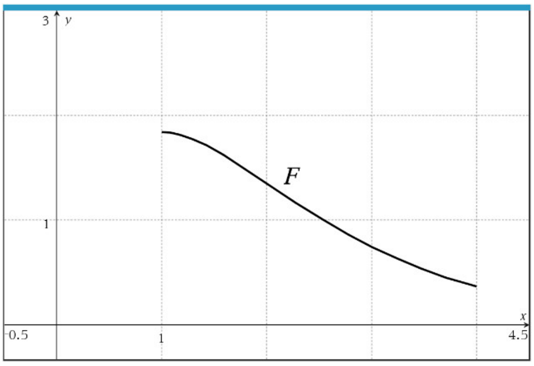
   Determine how many cents should be lost for the sum between 6 and 9 to make this game fair.

Exercise 2 Calc. : ✓

A kids' play area manufacturer wants to offer its customers a new model of slide. They create a diagram of the proposed slide in an oblique projection:



The profile of this slide is measured in meters and can be modeled by the function $F(x) = (ax - b)e^{-x}$, for $1 \leq x \leq 4$, where a and b are two parameters. The function F was drawn below.



1. It is planned that the tangent to the function F at the point where $x = 1$ would be horizontal.
   Determine the value of the parameter b. — 3 marks
2. It is also planned that the top of the slide will be at 1.85 meters.
   Determine the value of the parameter a. — 2 marks

The profile of the wall is finally modeled by $F(x) = 5x \cdot e^{-x}$.

3. **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m^2. — 2 marks
4. **Determine** the point on the slide where the gradient is greatest. — 3 marks

Exercise 3 Calc. : ✓

Optical smoke detectors contain a photocell as an important component. A factory produces photocells for this purpose. A controller automatically checks photocells and rejects those that are faulty. On average he is 86% accurate. However, the accuracy of the controller is found to vary — sometimes he detects a higher percentage of faulty photocells and sometimes a lower percentage. The controller's accuracy is found to be modelled by a normal distribution with a standard deviation of 5%.

1. **Find** the probability that the controller is less than 85% accurate. — 1 mark
2. $\frac{9}{10}$ of the time the controller is less than x% accurate. **Determine** x. — 2 marks
3. Given that, on a particular day, the controller is less than 90% accurate, **find** the probability that he is more than 85% accurate. — 2 marks

Two types of optical smoke detector are being tested for reliability. The higher the probability of an alarm being triggered the more reliable it is.
Type A contains a single photocell and is triggered when this photocell is activated.
Type B contains three photocells and is triggered if at least two of the three photocells are activated.
The probability of a photocell being activated in the presence of smoke is p. The probability of both types of alarm being triggered is calculated for different values of p.
$P\left(A_p\right)$ is the probability of type A being triggered when the probability is p,
$P\left(B_p\right)$ is the probability of type B being triggered when the probability is p.

4. **Complete** the table below. — 4 marks

p	0.3	0.5	0.7
$P\left(A_p\right)$	0.3	0.5	0.7
$P\left(B_p\right)$			
More reliable type			

5. **Determine** for what value of p does type B become more reliable than type A. — 2 marks
6. **Show** that, in terms of p, $P\left(A_p\right) = p$ and $P\left(B_p\right) = -2p^3 + 3p^2$. — 4 marks
7. **Explain** the meaning of the following function R in relation to the context of the question. **Explain** what is calculated in lines (1) to (3) and **interpret** the result. — 3 marks

$$R : p \mapsto R(p) = -2p^3 + 3p^2 - p$$
$$(1)\ R'(p) = -6p^2 + 6p - 1$$
$$(2)\ R'(p_1) = 0 \Rightarrow p_1 \approx 0.79$$
$$(3)\ R''(p_1) < 0$$

Exercise 4 Calc. : ✓

Given are the plane $E : 2x_1 - x_2 + 3x_3 = 5$ and for each $a \in \mathbb{R}$ a straight line:

$$g_a : \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ a \\ 2 \end{pmatrix}$$

1. **Determine** the coordinates of the intersection of the straight line g_a with the plane E in terms of a. — 4 marks
2. **Find** for which value of a is there no solution. — 3 marks

   Interpret the result geometrically.