Exercise 1 Calc. : ✗

Let f and g be two functions defined by:

$$f(x) = a + e^{-x+1} \qquad g(x) = \frac{b \cdot x + 2}{x - 1}$$

where a and b are real numbers.
Find the values of a and b such that f and g have the following properties:

- f and g have the same limit in $+\infty$.
- The graphs of functions f and g intercept in a point with abscissa 2.

5 marks

Exercise 2 Calc. : ✗

Consider vectors $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} n \\ 1 \\ -3 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$, where n is a real number.
Prove that whatever the value of n, the volume of the parallelepiped determined by these vectors is always the same.

5 marks

Exercise 3 Calc. : ✗

Solve the equation:

$$\log_2(x) + \log_2(x - 1) = 1$$

5 marks

Exercise 4 Calc. : ✗

Consider function f defined by $f(x) = x^2 \cdot \cos x$.
Of the four functions below, which one is a primitive function of f? Explain you answer.

$$F(x) = \frac{x^3}{3} \cdot \sin x \qquad H(x) = 2x \cdot \cos x + (x^2 - 2) \cdot \sin x$$

$$G(x) = -2x \cdot \sin x \qquad K(x) = 2x \cdot \cos x - x^2 \cdot \sin x$$

5 marks

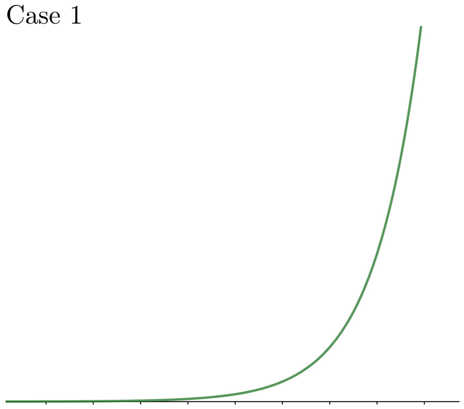
Exercise 5 Calc. : ✗

Let a and b be two non-zero real numbers and f be the function defined over $\mathbb{R}$ by:

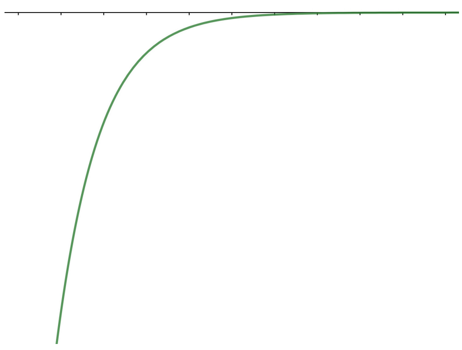
$$f(x) = a \cdot e^{b \cdot x}$$

Here are two possible shapes for the curve of this function.
In each case, give the possible values for a and b.

Case 1



Case 2



5 marks

Exercise 6 Calc. : ✗

Find a complex number z that is a cube root of $-8i$ and a fourth root of $-8 - 8i\sqrt{3}$.

5 marks

Exercise 7 Calc. : ✗

The Corbett Nation Park reserve in India is a natural reserve where we can see tigers.	
1. This reserve is home to 8 tigers, five of which are marked. We capture three tigers, what is the probability that two of them be marked? Give the result as an irreducible fraction.	2 marks
2. A group of 8 tourists arrives on the site for a safari. Four of these tourists must get into the first car, that has four different places. How many different ways can they fit in the car?	2 marks
3. We know that 40% of visitors to Corbett Nation Park are European. Among Europeans, 10% see a tiger. We also know that 20% of visitors to this reserve see a tiger. We come across a non-European visitor at random. Calculate the probability that he saw a tiger.	2 marks
4. Every day, the probability that a tourist sees a tiger is of 0.2.	
(a) Calculate the probability that a tourist sees a tiger for the first time on the third day of his visit.	2 marks
(b) We note $P(X = n) = p_n$ the probability that a tourist sees a tiger for the first time on the n-th day of his visit. Show that the sequence (p) is a geometric sequence of which we will specify the first term and reason.	2 marks
(c) Show that $P(X \leq n) = 1 - 0,8^n$. Interpret this result in this context.	3 marks

Exercise 8 Calc. : ✗

Let f and g be two functions defined by $$f(x) = -\frac{1}{2}\left(e^{2x} + e^{-2x}\right) \qquad g(x) = x^n \cdot \ln(x)$$ where n is a positive integer. Prove that the graphs of these two functions never intersect, whatever the value of n.	7 marks