Exercise 1 Calc. : ✓

Part 1

20 carp are introduced into an artificial lake. The pond has limited resources and the carp population is modelled by the function N, defined by $N(t) = \frac{200}{1 + k \cdot 2^{-t}}$,
where t is the time expressed as a whole number of years and k is a real number parameter. Carp only lay their eggs once every twelve months.

a) Based on the information given in the introduction, **verify** that $k = 9$. — 2 marks

b) **Determine** how long it takes for the population to exceed 90 individuals. — 2 marks

c) **Calculate** the carp population after 15 years and after 20 years. — 3 marks

Describe how the population is developing over a long period.

Part 2

The average length of tetra fish in a freshwater pond is well modelled by a normal distribution of mean $\mu = 8$ cm and standard deviation of $\sigma = 2$ cm.

d) **Calculate** the probability, that a tetra fish chosen at random from the pond has a length:

i) greater than 8 cm, — 2 marks

ii) between 6 cm and 8 cm. Round your result to three decimal places. — 2 marks

e) The probability a tetra fish chosen at random from the pond has a length of more than 6 cm is given as 0.84.

There are currently 65 tetra fish in the pond.

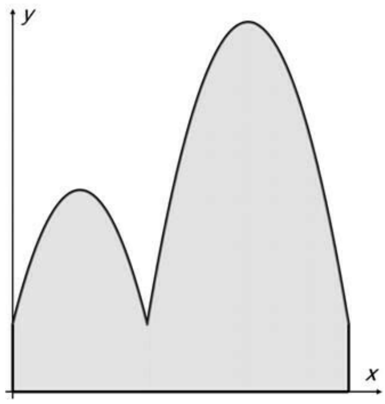
Calculate the probability that fewer than 50 of these fish are more than 6 cm long. — 2 marks

Part 3

The design for the surface of a freshwater pond at a trout breeding centre is represented by the shaded region in the figure. The edges of the region follow:

- a parabola, on the left, with equation $y = -x^2 + 4x + 2$,
- a parabola, on the right, with equation $y = -x2 + 14x - 38$,
- the x and y-axes and the line $x = 10$.

The units of measure for both x and y are metres.



f) **Determine** the area of the surface of the pond. — 4 marks

Part 4

The shellfish fishing sector in Italy suffered a decline in catches between 2010 and 2019, as recorded in the following table:

x: number of years since 2010	0	1	2	3	4	5	6	7	8	9
y: mass of shellfish caught, in tonnes	235	230	220	200	194	190	185	177	175	172

g) **Draw** a scatter diagram representing the data in the table, **interpret** the diagram and **describe** the correlation. — 4 marks

h) **Determine** an equation, in the form $y = mx + b$ of the linear regression of y on x and use this model to **estimate** the mass of shellfish caught in 2020. — 4 marks

Exercise 2 Calc. : ✓

Part 1

A survey was carried out among holidaymakers on their sporting activities during their holidays. The survey revealed that 45% of holidaymakers visited a gym during their holidays, and, of these, 60% went swimming.
Among holidaymakers who did not go to a gym, 70% went swimming.
A holidaymaker is chosen at random.
Consider the following events:
G: "the holidaymaker went to a gym";
S: "the holidaymaker went swimming".

a) **Construct** a tree diagram describing the situation. — 2 marks

b) **Describe** in words the event $G \cap S$. — 1 mark

c) **Show** that $P(S) = 0.655$. — 2 marks

d) **Calculate** the probability that a holidaymaker chosen at random went to a gym given that he/she went swimming. Round the result to four decimal places. — 2 marks

Four holidaymakers were chosen at random.
Let X be the random variable that gives the number of these holidaymakers swimming during their holiday.
As the number of holidaymakers is sufficiently large, we assume that the random variable follows a binomial distribution.

e) **Calculate** the probability that: — 5 marks

i) exactly two of the holidaymakers went swimming during their holiday,

ii) at least three of the holidaymakers went swimming during their holiday.

Round your answer to three decimal places.

Part 2

Some people have booked a tennis course during their holiday.
While waiting to start, they watch a cannon firing tennis balls that rise slightly as they leave the cannon and then pass over the middle of the court.
The height in metres of the centre of the ball expressed in terms of the time t seconds after leaving the cannon can be modelled by the function h, defined by:

$$h(t) = -4.9t^2 + 4.2t + 0.5,$$

The radius of a tennis ball is 3.4 cm.

f) **Determine** the time in seconds at which the ball will hit the ground, noting that the centre of the ball will be 3.4 cm above the ground. Round your answer to two decimal places. — 2 marks

g) **Determine** the maximum height reached by the centre of the ball. — 2 marks

The height in metres of the centre of the ball can also be expressed in terms of the horizontal distance, x metres, travelled by the centre of the ball after leaving the cannon, and can be modelled by the function f, defined by:

$$f(x) = -0.00784x^2 + 0.168x + 0.5,$$

In the middle of the court, the top of the net is 0.9 metres above the ground.
The net is 11.88 metres horizontally from the cannon.

h) **Show** that the ball will pass over the net. — 2 marks

The distance from the cannon to the opposite end of the court is 23.76 metres.

i) **Show** that the model predicts that the ball would not touch the ground within the court. — 2 marks

The quadratic model given above takes no account of air resistance. An adjusted model is proposed with the function g defined by: $$g(x) = -0.00784x^2 + 0.168x + 0.5 - 0.0003x^3,$$ where x and $g(x)$ are measures defined in the same way as x and $f(x)$.	
j) **Determine** whether the gradient at which the ball leaves the cannon is the same for both f and g.	2 marks
k) **Determine** whether the new function, g, would provide a model for path of the ball leaving the cannon at 50 cm height and landing within the opposite end of the court.	3 marks