Exercise 1
Calc. : X
We give three logarithmic functions $f, g$ and $h$ defined respectively by

$$
f(x)=3-4 \ln (x-2), \quad g(x)=3-\ln (x) \quad \text { et } \quad h(x)=\ln (x+3)
$$

as well as four graphs of logarithmic functions depicted below.


Associate each of the three functions with the corresponding graph. Justify the answer.

## Exercise 2

The shape of the bottom of a lake can be modeled by the function $f$ defined by

$$
f(x)=x^{3}-6 x^{2}, \quad 0 \leq x \leq 6
$$

where $x$ is measured in kilometers and $f(x)$ in meters.


Determine the depth of the lake at its deepest point.
Exercise 3
We know that the function $F$ defined by $F(x)=x^{2}+2 x$ is a primitive of the function $f$ and that $\int_{1}^{a} f(x) \mathrm{d} x=5$, where $a$ is a positive real number.
Determine $a$.

We consider the function $f$ defined by $f(x)=\frac{x^{2}}{2}$.
We seek to determine the area $A$ of the surface delimited by the graph of $f$, the $x$-axis and the lines of equations $x=0$ and $x=5$. (see diagram below left)


a) Using the rectangles represented (see diagram above at right), determine a lower and an upper bound of the area $A$ sought.
b) Explain how to obtain finer bounds with this technique.
c) Show that the function $F$ defined by $F(x)=\frac{x^{3}}{6}$ is an antiderivative of the function $f$ and calculate the exact value of the area $A$.

1 mark
1 mark

## Exercise 5

Calc. : $x$
The Ferris wheel of an amusement park has a diameter of 50 meters. It makes a complete turn uniformly every 120 seconds. Its highest point is located 55 meters from the ground. We consider the movement of a basket of the Ferris wheel. This is a periodic movement which can be modeled by a function $f$ defined by

$$
f(t)=a \cdot \sin (b(t-c))+d
$$

where $t$ represents the time, in seconds, and $f(t)$ the height of the gondola, in meters.
The basket is at the lowest point of the Ferris wheel at the moment $t=0$.
a) Show that the amplitude of the movement is equal to 25 meters.
b) Show that the vertical displacement is equal to 30 meters.
c) Show that $b=\frac{\pi}{60}$.
d) Determine $c$ and interpret the result.

## Exercise 6

Calc. : $X$
During a 100 m race, athlete Ali is placed in lane 3 on the starting line.
There are 8 corridors in total.
Three other athletes taking part in the race are placed in the other corridors.
Calculate the probability that none of the other three runners are placed at Ali's side.
5 marks

## Exercise 7

Lactase is an enzyme that helps digest dairy products. containing lactose.
$30 \%$ of the world's population produces its own lactase in intestines.
A medical company has developed a new rapid test to see whether or not an individual produces lactase. A study is carried out to determine the reliability of the rapid test and the study shows that this test gives a correct positive result for $80 \%$ of individuals who produce lactase but gives a false positive for $10 \%$ of individuals who do not produce lactase.
The rapid test is used on a large population.
a) Show that $69 \%$ of the population should get a negative result.
b) Calculate the probability that an individual produces lactase, knowing that he obtained a negative result.

2 marks

## Exercise 8

Calc. : $x$
Andy is a basketball player. The probability that he succeeds a free throw is $75 \%$. He is allowed 3 attempts and each successful throw is worth one point.
a) Does the number of points scored follow a binomial law? Justify the answer.

3 marks
b) Calculate the probability that Andy scores at least one point.

## Exercise 9

A company produces chocolate bars whose mass follows a normal law with mean $\mu=100 \mathrm{~g}$ and standard deviation $\sigma=1 \mathrm{~g}$.
We choose a chocolate bar at random from the production.
a) Determine the probability that this tablet weighs between 97 g and 103 g .

3 marks
b) Determine the probability that this tablet weighs more than 100 g .

## Exercise 10

After some complaints about the new schedules, management of one school claims that only $10 \%$ of teachers are unhappy with their new schedule. Some teachers think that it is more than $10 \%$. They then ask their opinion to a group of 35 teachers chosen at random.
An NHST test is carried out at a significance level of $5 \%$.
a) Determine if this test is one-sided left or right. Justify the answer.
b) Formulate an appropriate null hypothesis $H_{0}$ and an alternative hypothesis $H_{1}$ for this test.

The random variable $X$ designates the number of teachers unhappy with their new schedule in a sample of 35 teachers.
The table below shows the values of $P(X \geq k)$ with $k=1,2,3,4,56,7,8,9$ and 10 assuming that $10 \%$ of teachers are unhappy.

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(X \geq k)$ | 0.975 | 0.878 | 0.694 | 0.469 | 0.269 | 0.132 | 0.055 | 0.020 | 0.006 | 0.002 |

c) Determine the critical value $k$ and interpret this value.

Calc. : $X$

1 mark
1 mark

3 marks

