MATHEMATICS 3 PERIODS
PART A

RESERVE Questions

DATE : 18 June 2024, afternoon
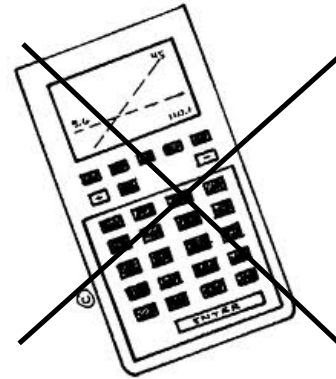
DURATION OF THE EXAMINATION:

2 hours (120 minutes)

AUTHORISED MATERIAL:

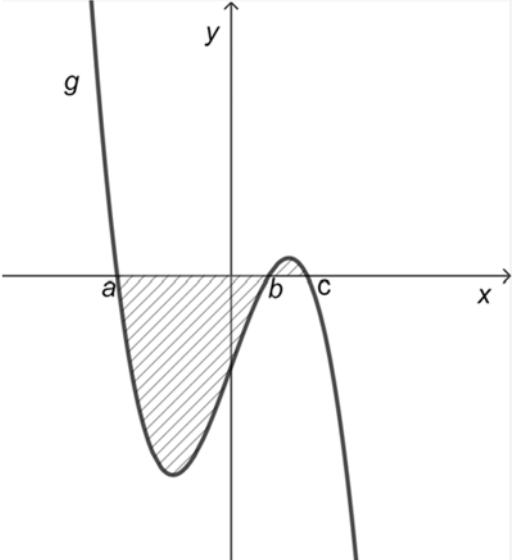Examination without technological tool
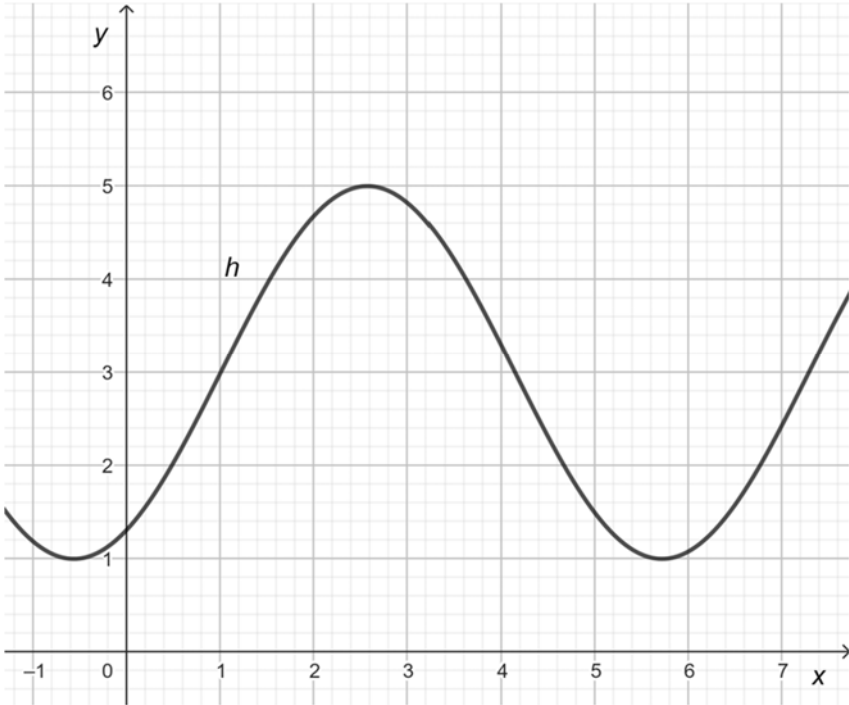
Pencil for the graphs

Formelsammlung / Formula booklet / Recueil de formules

SPECIFIC INSTRUCTIONS:

- Answers must be supported by explanations.
- Answers must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.

- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.

- When the answer provided is not the correct one, some marks can be awarded if it is evident that an appropriate method and/or a correct approach has been used.

PART A	Page 1/6	Marks

1) Consider the function f defined by

$$f(x) = 2x^3 - 9x^2 + 45.$$

Determine the coordinates of the extrema of the function f and **specify** their nature. — 5 marks

2) The diagram below shows the graph of a function g.

Specify for each of the following expressions, whether it represents the hatched area. **Justify** your answer. — 5 marks

a) $\int_a^c g(x)dx$

b) $\int_a^b g(x)dx + \int_b^c g(x)dx$

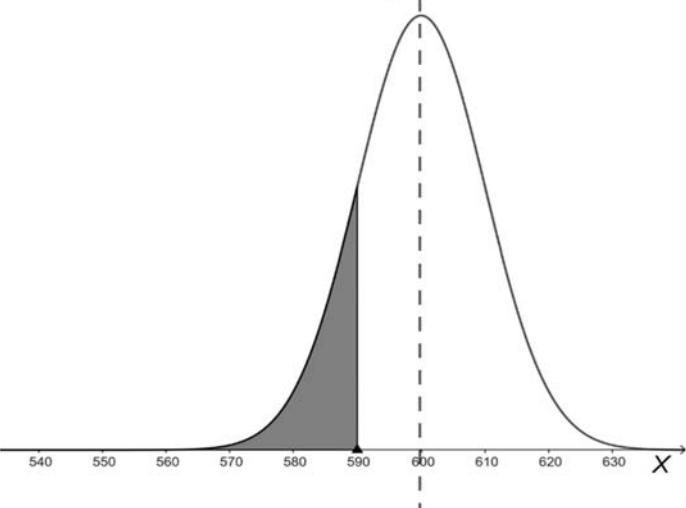c) $\left| \int_a^c g(x)dx \right|$

d) $-\int_a^b g(x)dx + \int_b^c g(x)dx.$

PART A	Page 2/6	Marks

3) A car is travelling along a horizontal road and the distance from the origin is observed over a period of 8 seconds, starting at $t = 4$ seconds.

The distance is given by the function d defined by

$d(t) = \dfrac{1}{4}t^3 - 2t^2 + 5t + 3$ with $t \in [4,12]$

where t is the time expressed in seconds, and $d(t)$ is expressed in metres.

a) **Show** that at the start of the observation, the car is 7 metres from the origin. — 1 mark

b) **Determine** the average velocity of the car between 4 seconds and 10 seconds. — 2 marks

c) **Determine** the instantaneous velocity of the car at $t = 10$ seconds. — 2 marks

4) The diagram below shows the graph of a sine function h with a period length of 2π.

Determine $h(x)$. — 5 marks

PART A	Page 3/6	Marks

5) Between 1st January 2000 and 1st January 2023, a group of scientists studied the surface area of a lake in a mountainous region of Europe. They created a model which suggests that the surface area of the lake decreases by 10% each year.

The initial surface area of the lake is 5 km².

    a) **Explain** why the surface area of the lake can be modelled by a function s defined by $s(t) = 5 \cdot 0.9^t$, where t is the number of years since year 2000 and $s(t)$ is expressed in km². — 2 marks

    b) Using this model, **determine** the surface area of the lake in 2002. — 1 mark

    c) Assume the model will remain valid over time. — 2 marks

       Describe the evolution of the surface area of the lake over time.

6) Peter applies for his first job. He sends his CV letter to 2 different companies.
The probability that exactly one company will reply to him is 0.45.
The probability that no company will reply to him is 0.3 .

    a) **Draw** a Venn diagram to illustrate the above information. — 2 marks

    b) **Determine** the probability that both companies will reply to Peter. **Give** the answer as a percentage. — 3 marks

PART A	Page 4/6	Marks

7) The distribution of peppers at a grower's market stand is as follows:

$\dfrac{2}{5}$ of peppers are green, of which half are organic.

$\dfrac{9}{20}$ of peppers are red, of which 40% are organic.

$\dfrac{3}{20}$ of peppers are yellow, of which 80% are organic.

A pepper is chosen at random.

Determine the probability that this pepper is organic.

5 marks

8) In a football team composed of 18 players, 3 are goalkeepers, 5 are defenders, 6 are midfielders and 4 are forwards.

a) The trainer chooses 3 of those defenders to play the next match.

Calculate how many different groups of three defenders the trainer can choose.

1 mark

b) The three defenders have been chosen. Now, one of them is assigned the left part of the field, one of them is assigned the central part, and one of them is assigned the right part.

Calculate in how many different ways those 3 defenders can position themselves on the field.

1 mark

c) 11 players are to be selected to play the game: this team will consist of 1 goalkeeper, 3 defenders, 5 midfielders and 2 forwards.
The 3 defenders have been chosen.

Determine how many different groups of 8 players the trainer can choose to fill the remaining places.

3 marks

PART A	Page 5/6	Marks

9) A brewery has a machine that fills bottles with soft drinks. The machine is set up so that the quantity of soft drink filled in a bottle is normally distributed with a mean of 505 ml and a standard deviation of 2 ml.

    a) **Determine** the probability that a randomly chosen bottle contains at least 505 ml of soft drink. — 1 mark

    b) **Determine** the probability that a randomly chosen bottle contains between 501 ml and 509 ml of soft drink. — 1 mark

Another machine fills bottles with juice.
It is assumed that the quantity of juice contained in a bottle follows a normal distribution with mean μ ml and standard deviation σ ml.
It is given that $P(X \leq 590) = 0.1587$.
The graph of this normal distribution is given below.

    c) **Give** the value of the mean of this normal distribution and **justify** your answer. — 1 mark

    d) **Determine** the probability that a randomly chosen bottle contains more than 590 ml of juice. **Give** the answer to the nearest tenth. — 2 marks

PART A	Page 6/6	Marks
10) In a population of fish, approximatively 42% are female. A test is conducted because it is possible that, in fact, this proportion is less.		
a) **State** the null hypothesis H_0 and the alternative hypothesis H_1.		2 marks
b) Let X be the random variable that gives the number of female fish in a sample of 20 fish. The table below shows values of $P(X \leq k)$ for $k = 3, 4, 5, 6, 7, 8,$ for a probability of 42% that a given fish is a female.		

k	3	4	5	6	7	8
$P(X \leq k)$	0.0102	0.0349	0.0922	0.1959	0.3461	0.5229

Determine the critical value k if the significance level is set at 5%, and **interpret** this value.		3 marks