

MATHEMATICS 3 PERIODS PART B

Reserve QUESTIONS

DATE: 18 June 2024 Morning

DURATION OF THE EXAMINATION:

2 hours (120 minutes)

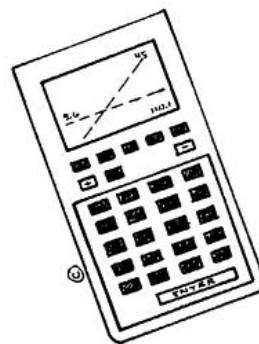
AUTHORISED MATERIAL:

Examination with technological tool:

Approved calculator

Pencil for the graphs

Formelsammlung/ Formula booklet/Recueil de formules

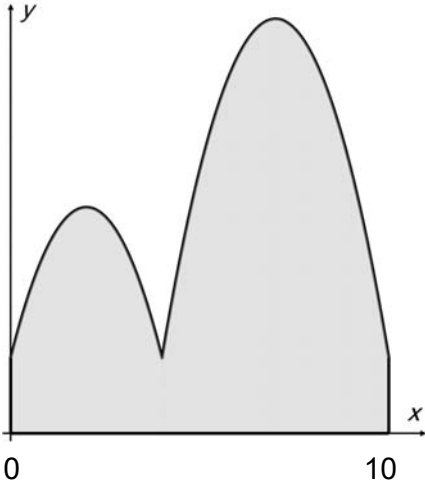


SPECIFIC INSTRUCTIONS:

- Use a different page for each question.
- Answers must be supported by explanations.
- Answers must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.

EUROPEAN BACCALAUREATE 2024: Reserve MATHEMATICS 3 PERIODS

PART B		
QUESTION B1	Page 1/2	Marks
<p>Part 1</p> <p>20 carp are introduced into an artificial lake. The pond has limited resources and the carp population is modelled by the function N, defined by</p> $N(t) = \frac{200}{1 + k \cdot 2^{-t}},$ <p>where t is the time expressed as a whole number of years and k is a real number parameter.</p> <p>Carp only lay their eggs once every twelve months.</p> <p>a) Based on the information given in the introduction, verify that $k = 9$.</p> <p>b) Determine how long it takes for the population to exceed 90 individuals.</p> <p>c) Calculate the carp population after 15 years and after 20 years. Describe how the population is developing over a long period.</p>		<p>2 marks</p> <p>2 marks</p> <p>3 marks</p>
<p>Part 2</p> <p>The average length of tetra fish in a freshwater pond is well modelled by a normal distribution of mean $\mu = 8$ cm and standard deviation of $\sigma = 2$ cm.</p> <p>d) Calculate the probability, that a tetra fish chosen at random from the pond has a length:</p> <p>i) greater than 8 cm,</p> <p>ii) between 6 cm and 8 cm. Round your result to three decimal places.</p> <p>e) The probability a tetra fish chosen at random from the pond has a length of more than 6 cm is given as 0.84 .</p> <p>There are currently 65 tetra fish in the pond.</p> <p>Calculate the probability that fewer than 50 of these fish are more than 6 cm long.</p>		<p>2 marks</p> <p>2 marks</p> <p>2 marks</p>

PART B																								
QUESTION B1	Page 2/2	Marks																						
<p>Part 3</p> <p>The design for the surface of a freshwater pond at a trout breeding centre is represented by the shaded region in the figure.</p> <p>The edges of the region follow:</p> <ul style="list-style-type: none"> • a parabola, on the left, with equation $y = -x^2 + 4x + 2$, • a parabola, on the right, with equation $y = -x^2 + 14x - 38$, • the x and y-axes and the line $x = 10$. <p>The units of measure for both x and y are metres.</p>		4 marks																						
<p>Part 4</p> <p>The shellfish fishing sector in Italy suffered a decline in catches between 2010 and 2019, as recorded in the following table:</p> <table border="1" style="width: 100%; border-collapse: collapse; margin: 10px 0;"> <tr> <td style="padding: 5px;">x: number of years since 2010</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">6</td> <td style="padding: 5px;">7</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">9</td> </tr> <tr> <td style="padding: 5px;">y: mass of shellfish caught, in tonnes</td> <td style="padding: 5px;">235</td> <td style="padding: 5px;">230</td> <td style="padding: 5px;">220</td> <td style="padding: 5px;">200</td> <td style="padding: 5px;">194</td> <td style="padding: 5px;">190</td> <td style="padding: 5px;">185</td> <td style="padding: 5px;">177</td> <td style="padding: 5px;">175</td> <td style="padding: 5px;">172</td> </tr> </table>	x: number of years since 2010	0	1	2	3	4	5	6	7	8	9	y: mass of shellfish caught, in tonnes	235	230	220	200	194	190	185	177	175	172	<p>g) Draw a scatter diagram representing the data in the table, interpret the diagram and describe the correlation.</p> <p>h) Determine an equation, in the form $y = mx + b$ of the linear regression of y on x and use this model to estimate the mass of shellfish caught in 2020.</p>	<p>4 marks</p> <p>4 marks</p>
x: number of years since 2010	0	1	2	3	4	5	6	7	8	9														
y: mass of shellfish caught, in tonnes	235	230	220	200	194	190	185	177	175	172														

EUROPEAN BACCALAUREATE 2024: Reserve MATHEMATICS 3 PERIODS

PART B		
QUESTION B2	Page 1/3	Marks
<p>Part 1</p> <p>A survey was carried out among holidaymakers on their sporting activities during their holidays. The survey revealed that 45% of holidaymakers visited a gym during their holidays, and, of these, 60% went swimming. Among holidaymakers who did not go to a gym, 70% went swimming.</p> <p>A holidaymaker is chosen at random.</p> <p>Consider the following events: G: “the holidaymaker went to a gym”; S: “the holidaymaker went swimming”.</p> <p>a) Construct a tree diagram describing the situation.</p> <p>b) Describe in words the event $G \cap S$.</p> <p>c) Show that $P(S)=0.655$.</p> <p>d) Calculate the probability that a holidaymaker chosen at random went to a gym given that he/she went swimming. Round the result to four decimal places.</p> <p>Four holidaymakers were chosen at random.</p> <p>Let X be the random variable that gives the number of these holidaymakers swimming during their holiday.</p> <p>As the number of holidaymakers is sufficiently large, we assume that the random variable follows a binomial distribution.</p> <p>e) Calculate the probability that:</p> <p style="margin-left: 40px;">i) exactly two of the holidaymakers went swimming during their holiday,</p> <p style="margin-left: 40px;">ii) at least three of the holidaymakers went swimming during their holiday.</p> <p>Round your answer to three decimal places.</p>		<p>2 marks</p> <p>1 mark</p> <p>2 marks</p> <p>2 marks</p> <p>5 marks</p>

PART B		
QUESTION B2	Page 2/3	Marks
<p>Part 2</p> <p>Some people have booked a tennis course during their holiday.</p> <p>While waiting to start, they watch a cannon firing tennis balls that rise slightly as they leave the cannon and then pass over the middle of the court.</p> <p>The height in metres of the centre of the ball expressed in terms of the time t seconds after leaving the cannon can be modelled by the function h, defined by:</p> $h(t) = -4.9t^2 + 4.2t + 0.5$ <p>The radius of a tennis ball is 3.4 cm.</p> <p>f) Determine the time in seconds at which the ball will hit the ground, noting that the centre of the ball will be 3.4 cm above the ground. Round your answer to two decimal places.</p> <p>g) Determine the maximum height reached by the centre of the ball.</p> <p>The height in metres of the centre of the ball can also be expressed in terms of the horizontal distance, x metres, travelled by the centre of the ball after leaving the cannon, and can be modelled by the function f, defined by:</p> $f(x) = -0.00784x^2 + 0.168x + 0.5.$ <p>In the middle of the court, the top of the net is 0.9 metres above the ground. The net is 11.88 metres horizontally from the cannon.</p> <p>h) Show that the ball will pass over the net.</p> <p>The distance from the cannon to the opposite end of the court is 23.76 metres.</p> <p>i) Show that the model predicts that the ball would not touch the ground within the court.</p>		<p>2 marks</p> <p>2 marks</p> <p>2 marks</p> <p>2 marks</p>

EUROPEAN BACCALAUREATE 2024: Reserve MATHEMATICS 3 PERIODS

PART B		
QUESTION B2	Page 3/3	Marks
<p>The quadratic model given above takes no account of air resistance. An adjusted model is proposed with the function g defined by: $g(x) = -0.00784x^2 + 0.168x + 0.5 - 0.0003x^3.$ where x and $g(x)$ are measures defined in the same way as x and $f(x)$.</p>		
<p>j) Determine whether the gradient at which the ball leaves the cannon is the same for both f and g.</p>		2 marks
<p>k) Determine whether the new function, g, would provide a model for path of the ball leaving the cannon at 50 cm height and landing within the opposite end of the court.</p>		3 marks