S7ICT.
Graphs

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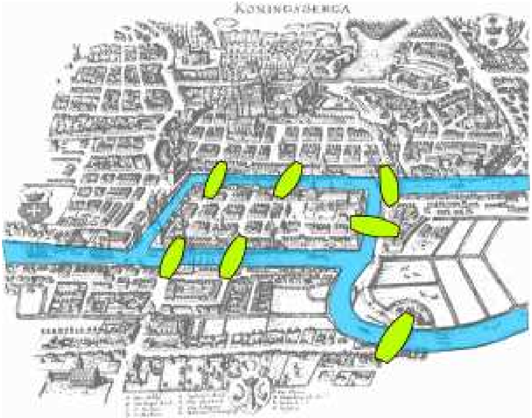
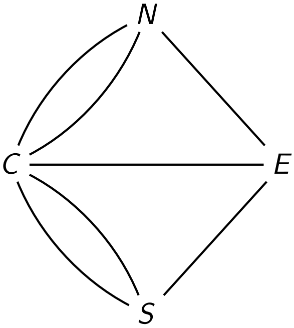
Chapter outlook

- Examples of graphs
- (Some) graph theory
- A special type of graphs : trees

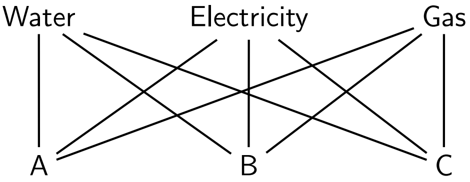
A first example : the Königsberg bridges





Source : Wikimedia Commons.

1736 : Euler wants to travel through the 7 bridges of this city without travelling twice on the same bridge.



Is it possible to draw the 9 pipes from each utility to each house without making them cross ?

Planar graphs : 4 colors only

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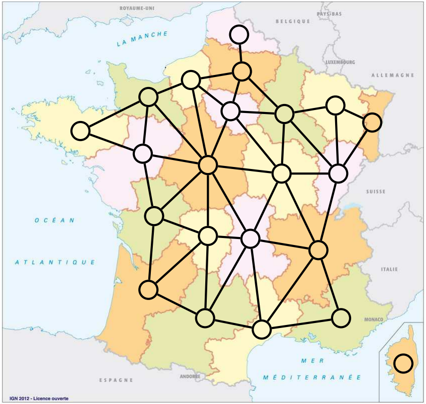
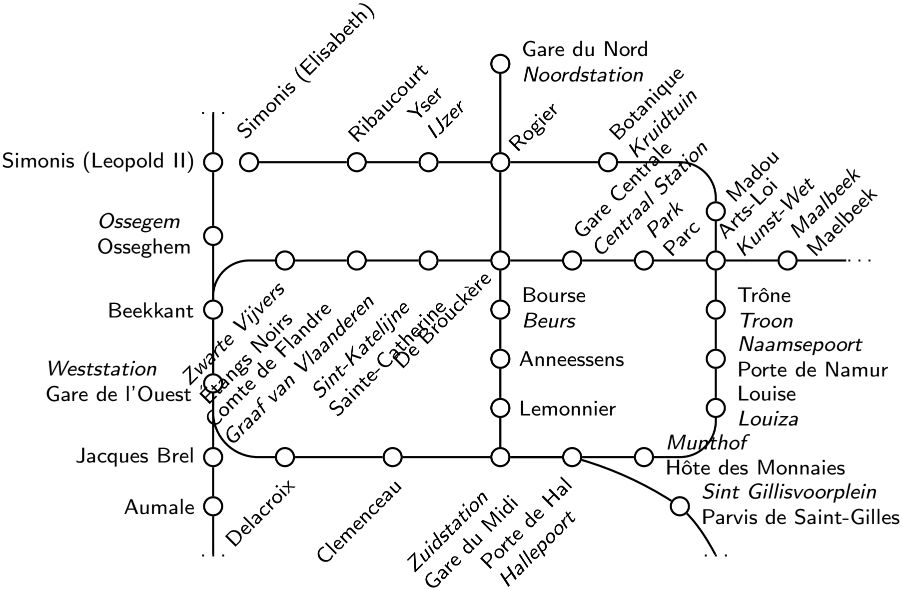


The associated graph.

Extract from Brussels subway map



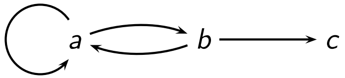
A simple, oriented graph is given by a set V (the vertices) and a set $E \subset V \times V$ (the edges).

Later on, we'll use $n = |V|$ (number of vertices) and $m = |E|$ (number of edges).

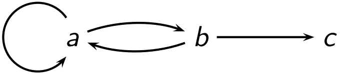
Here is an "abstract" example to illustrate :

- $V = \{a; b; c\}$
- $E = \{(a, a); (a, b); (b, a); (b, c)\}$

It's a graph with 3 vertices and 4 edges. Let's draw it !



Graphs : alternate representations



1. Successors table : for each vertex i, the set of vertices j for which the edge (i,j) exists, i.e. $\Gamma^+(i) = \{j \in V | (i,j) \in E\}$.

Vertex	a	b	c
Successors	a, b	a, c	$-$

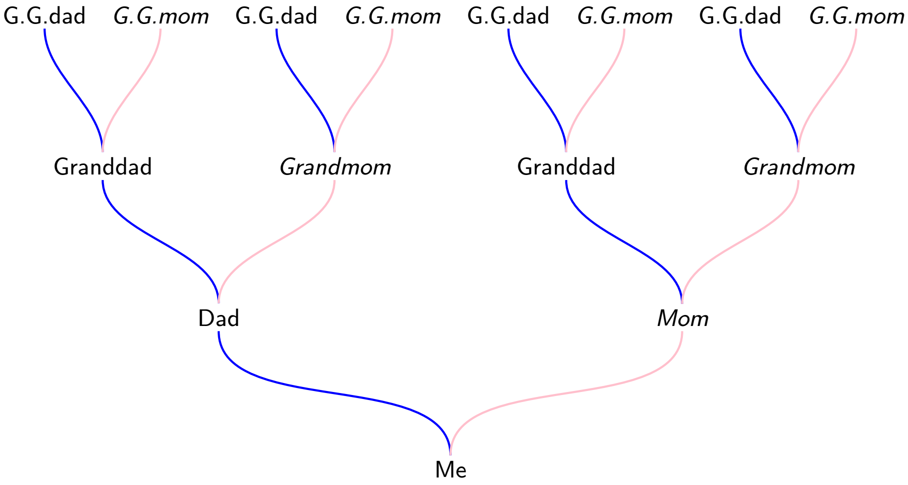
2. Predecessors table : for each vertex i, the set of vertices j for which the edge (j,i) exists, i.e. $\Gamma^-(i) = \{j \in V | (j,i) \in E\}$.

Vertex	a	b	c
Predecessors	a, b	a	b

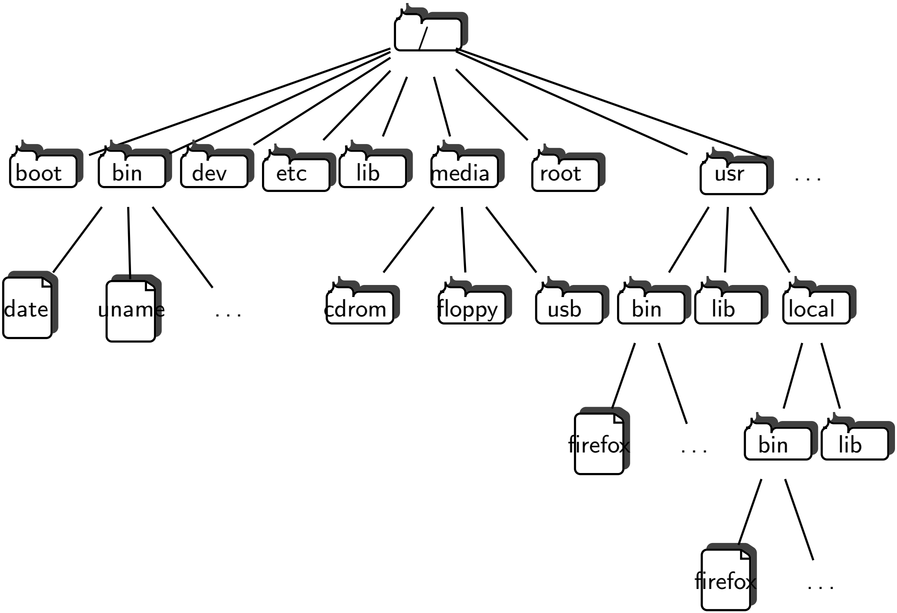
3. Adjacency matrix : given an order on vertices (e.g., the lexicographical order), it is the matrix M (a 2d array of size $n \times n$) where :

$$(M)_{i,j} = \begin{cases} 1 & \text{when } (i,j) \in E \\ 0 & \text{when it's not} \end{cases} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Trees : genealogy tree



Trees : file tree



Trees : where are they ?