Question B1

B1 Math 3p 2014

The function *fab* is given by

$\mathbf{fab}(x):=x^3+a\cdot x^2+x+b$ ▸ *Udført*

a) The derivative *fp* is given by

$\mathbf{fp}(x):=\frac{d}{dx}(\mathbf{fab}(x))$ ▸ *Udført*

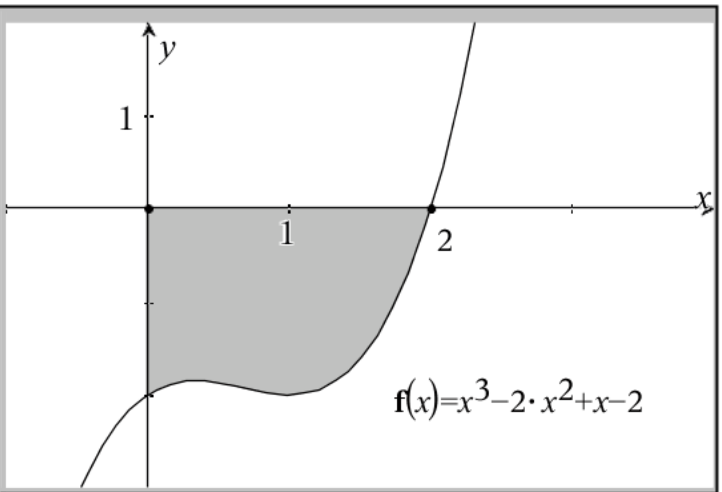
Is is given that fab(0)=−2 and fp(1)=0. We solve these equations for *a* and *b*:

$\text{solve}\left(\begin{cases}\mathbf{fab}(0)=^-2\\ \mathbf{fp}(1)=0\end{cases},a,b\right)$ ▸ $a=^-2$ and $b=^-2$

∴ a = −2 and b = −2

b) $\int_0^2 \mathbf{f}(x)\,dx=\frac{^-10}{3}\approx -3.33$

The absolute value of this result is the area of the region bounded by the graph and the coordinate axes, see graph.



c) The volume V of the solid of revolution generated by the region in b) is

$V=\pi\cdot\int_0^2 (\mathbf{f}(x))^2\,dx=\frac{632\cdot\pi}{105}\approx$

$\frac{632\cdot\pi}{105}$ ▸ 18.9094 .

Question B2

B2 Math 3p 2014

The curve formed by the rope is given by

$y=\mathbf{fc}(x):=c\cdot(\mathbf{e}^x+\mathbf{e}^{-x})$

a) It is given that fc(±4)=3.

$\text{solve}(\mathbf{fc}(4)=3,c)$ ▸ $c=0.054928$.

Hence c ≈ 0.055, q.e.d.

We define $\mathbf{f}(x):=0.055\cdot(\mathbf{e}^x+\mathbf{e}^{-x})$**.**

b) The *x*−value of the lowest point is 0, which can also be found by using the fMin−command

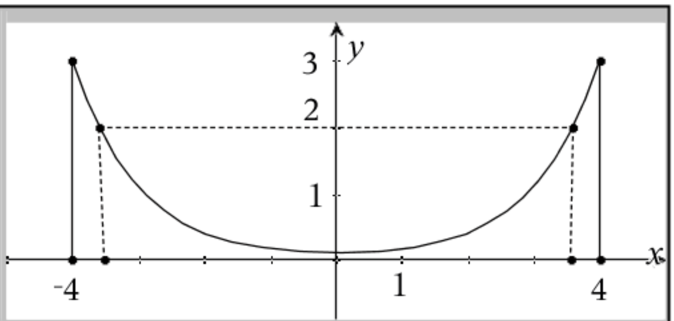
$\text{fMin}(\mathbf{f}(x),x)$ ▸ $x=0$.

Hence the lowest height is $\mathbf{f}(0)$ = 0.11 or 11 cm.

c) Two points 2 meters above the ground:

$\text{solve}(\mathbf{f}(x)=2,x)$ ▸ $x=^-3.59281$ or $x=3.59281$

Hence the distance between the points is
2·3.59281 meters ≈ **7.2 meter**



c)

$\text{arcLen}(\mathbf{f}(x),x,^-4,4)$ ▸ 10.8431

or

$\int_{-4}^{4}\sqrt{1+\left(\frac{d}{dx}(\mathbf{f}(x))\right)^2}\,dx$ ▸ 10.8431

Hence the length of the rope is 10.8 meters

Question B3

B3 Math 3p 2014

a)

P(both def) = P(A∩B) = $0.02 \cdot 0.01 = 0.002$

P(at least one defect) = P(A∪B) =

$0.02 + 0.01 - 0.002 = 0.0298$

∴ P(no defect) = 1 − 0.0298 = 0.9702

b) $P(A\cap B|\text{defect}) = \dfrac{P(A\cap B)}{P(def)} = \dfrac{0.0002}{0.0298} = \mathbf{0.00671}$

c) X = # not defective out of 100

$P(X \geq 95) =$

binomCdf(100,0.97,95,100) = 0.919163 ≈ **0.92**

d) Y = # not defective out of n

$p(n)$:=binomCdf(n,0.97,50,n)

$p(52) = 0.795393$

Hence it is not enough to order 52 chips.

We try with bigger orders:

p$(54) = 0.977236$; 54 not enough

$p(55) = 0.993994$; 55 not enough

$p(56) = 0.998602$; 56 ok.

Hence the minimum number to order is 56 chips.

e) Mass m normally distributed with $\mu = 500$ and $\sigma = 4$.

$P(490 \leq m \leq 510) =$

normCdf(490,510,500,4) = 0.987581

Hence 98.8 % of the chips have weight between 490 mg and 510 mg.

Question B4

B4 Math 3p 2014

About the spreadsheet below:

Given data in columns A (= x) and B (= y) .

Linear regression of y on x in column C and D.

$z = \ln(y)$ calculated in column E.

Linear regression of z on x in column F and G.

	A year	B tons	C	D	E zzz	F	G	H	I
=				=LinRegM	=1.*ln(tons)		=LinRegM		
1	1	65	Titel	Lineær r...	4.17439	Titel	Lineær r...		
2	2	76	RegEqn	m*x+b	4.33073	RegEqn	m*x+b		
3	3	119	m	79.8571	4.77912	m	0.407119		
4	4	162	b	⁻81.6667	5.0876	b	3.60126		
5	5	260	r²	0.807511	5.56068	r²	0.968205		
6	6	505	r	0.898616	6.22456	r	0.983974		
7			Resid	{66.8095..		Resid	{0.16600..		

A1 1

a) Equation of the regression line of y on x

y = $\mathbf{f1}(x) = 79.9 \cdot x - 81.7$

b) See spreadsheet above.

c) Equation of the regression line of z on x

$z = \mathbf{f2}(x) = 0.407 \cdot x + 3.6$

d) To deduce the new relation between y and x we solve the equation f2(x) = z = ln(y) for y :

$y = \mathbf{e}^{\mathbf{f2}(x)}$ ▸ $\boldsymbol{y = 36.6445 \cdot (1.50248)^x}$

e) Linear model 2014 (x = 7): $\mathbf{f1}(7) \approx 477$ tonnes.

Exponential model: $\mathbf{e}^{\mathbf{f2}(7)} = 633.397 \approx 633$ tonnes.

f) **The exponential model is clearly the better (see graph).**

solve$\left(\mathbf{e}^{\mathbf{f2}(x)} > 3000, x\right)$ ▸ $x > 10.8202$

Check: $\mathbf{e}^{\mathbf{f2}(10)}$ ▸ 2148.35 ; $\mathbf{e}^{\mathbf{f2}(11)}$ ▸ 3227.86

The production exceeds 3000 tonnes in 2018.
(2007+11)