Math 3p 2015 Question B1

$\mathbf{f}(x):=0.75\cdot x^3-1.25\cdot x^2-1$ ▸ *Done*

$\mathbf{g}(x):=x^2-1$ ▸ *Done*

a) $\text{solve}(\mathbf{f}(x)=\mathbf{g}(x),x)$ ▸ $x=0.$ or $x=3.$

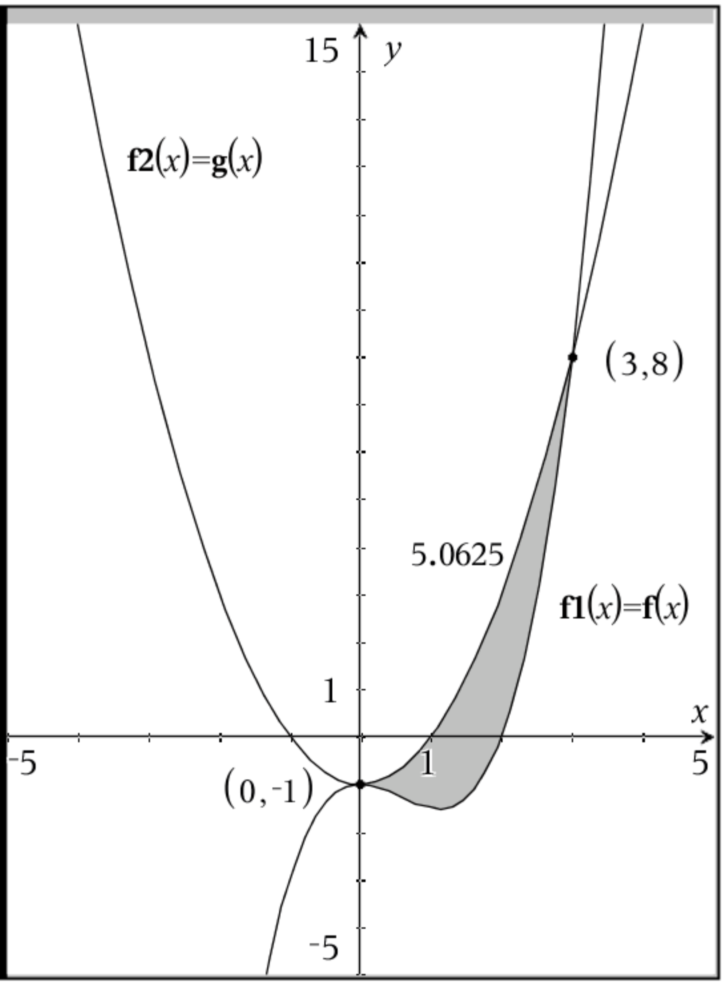
$\mathbf{f}(0)=^{-}1.\quad \mathbf{f}(3)=8.$

Points of intersection (0,−1) and (3,8).

b) $\int_0^3 (\mathbf{g}(x)-\mathbf{f}(x))\mathrm{d}x = 5.0625$ (or $\mathbf{\frac{81}{16}}$)

c) $\int_0^3 \sqrt{1+\left(\frac{d}{dx}(\mathbf{f}(x))\right)^2}\,\mathrm{d}x = \mathbf{11.1663}$ or

$\text{arcLen}(\mathbf{f}(x),x,0,3) = \mathbf{11.1663}$



Math 3p 2015 Question B2

$\mathbf{f}(t):=9\cdot \boldsymbol{e}^{-0.12\cdot t}+11$ ▸ *Done*

a) $\mathbf{f}(2)=18.0797$ and $\mathbf{f}(9)=14.0564$**.** Hence

Midnight temperature 18 °C and at 7:00 next morning 14 °C.

b) $\mathbf{fp}(t):=\frac{d}{dt}(\mathbf{f}(t))$ ▸ *Done* $\quad\mathbf{fp}(t)$ ▸ $^{-}1.08\cdot(0.88692)^t$, i.e. $\boldsymbol{f'(t)=^{-}1.08\cdot(0.88692)^t}$

$\boldsymbol{f'(t)=^{-}1.08\cdot(0.88692)^t}$ is negative for all t, hence f is decreasing.

c) $\mathbf{fp}(2)=^{-}0.85$, hence **temperature decreases by 0.85 °C per hour at midnight (t=2).**

d) $\text{solve}(\mathbf{f}(t)=15,t)$ ▸ $t=6.75775$

Temperature falls below 15°C at t=6.76. i.e. at 4:45:28 in the morning.

e) $\int_0^9 \left(0.7\cdot \boldsymbol{e}^{-0.12\cdot t}\right)\mathbf{d}t = 3.85$.

Hence **3.85 kWh is going out of the room between 22:00 h (t=0) and 7:00h (t=9)**

Math 3p 2015 Question B3

a) **Expected number of vegetarian menus: $120 \cdot 0.2 = 24$.**

b) X = # of vegetarian menus chosen is distributed binom(120,0.20).

$P(X \le 28) = \text{binomCdf}(120,0.2,0,28) = 0.8477$

Probability that enough vegetarian menus were prepared is 0.8477.

c) X = # of vegetarian menus chosen is distributed binom(120,0.20).

The number of vegetarian menus prepared is called v.

$v=31$: $\text{binomCdf}(120,0.2,0,31) = 0.952975$

$v=32$: $\text{binomCdf}(120,0.2,0,32) = 0.970425$

$v=33$: $\text{binomCdf}(120,0.2,0,33) = 0.982058$

At least 33 vegetarian menus must be prepared.

d) Y=#of fish menus chosen is normally distributed with mean $\mu=240$ and standard deviation σ. Two ways of finding σ:

Given: $P(200 \le Y \le 280) = 0.95$,

i.e. $4 \cdot \sigma \approx 80$ or $\boldsymbol{\sigma \approx 20}$

Or use solve with a guess:

$\text{solve}(\text{normCdf}(200,280,240,\sigma)=0.95, \sigma=20)$

▸ $\sigma=20.4086$ ⚠

Hence $\boldsymbol{\sigma=20.4}$

e) $P(Y>320)=$
$\text{normCdf}(320,\infty,240,20) = 0.000032$

Very unlikely.

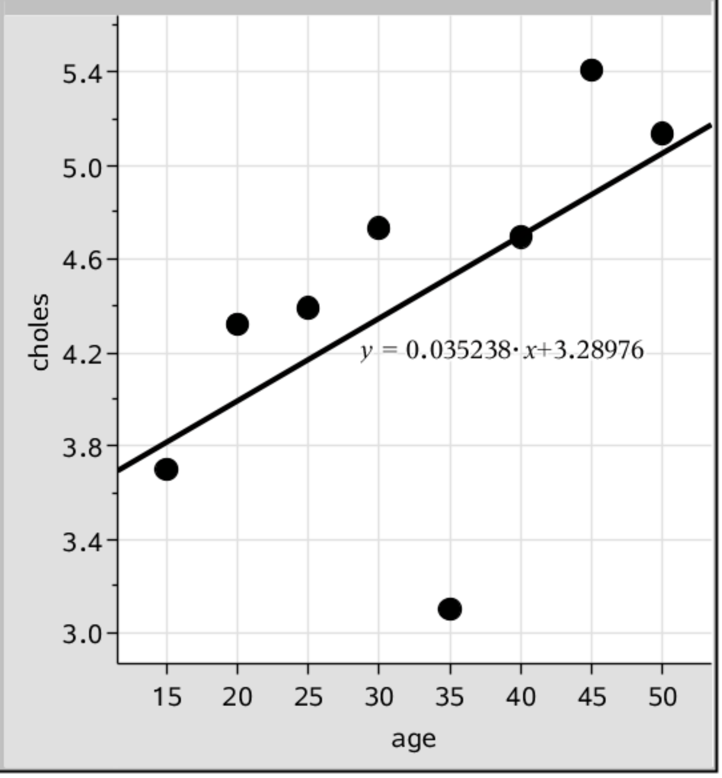
Another argument: 320 is 4 standard deviations above the mean.

Math 3p 2015 Question B4

The diagram below shows the scatter plot, demanded in a), and the regression line, demanded in b).

	A age	B choles	C	D	E
=				=LinRegM	
1	15	3.7	Title	Linear R...	
2	20	4.32	RegE..	m*x+b	
3	25	4.39	m	0.035238	
4	30	4.73	b	3.28976	
5	35	3.1	r²	0.331135	
6	40	4.69	r	0.575443	
7	45	5.41	Resi...	{⁻0.1183...	
8	50	5.14			
9					

D8



b) See spreadsheet and diagram above.

Regression line

$y = \mathbf{f1}(x)$ ▸ $\mathbf{y = 0.035 \cdot x + 3.29}$

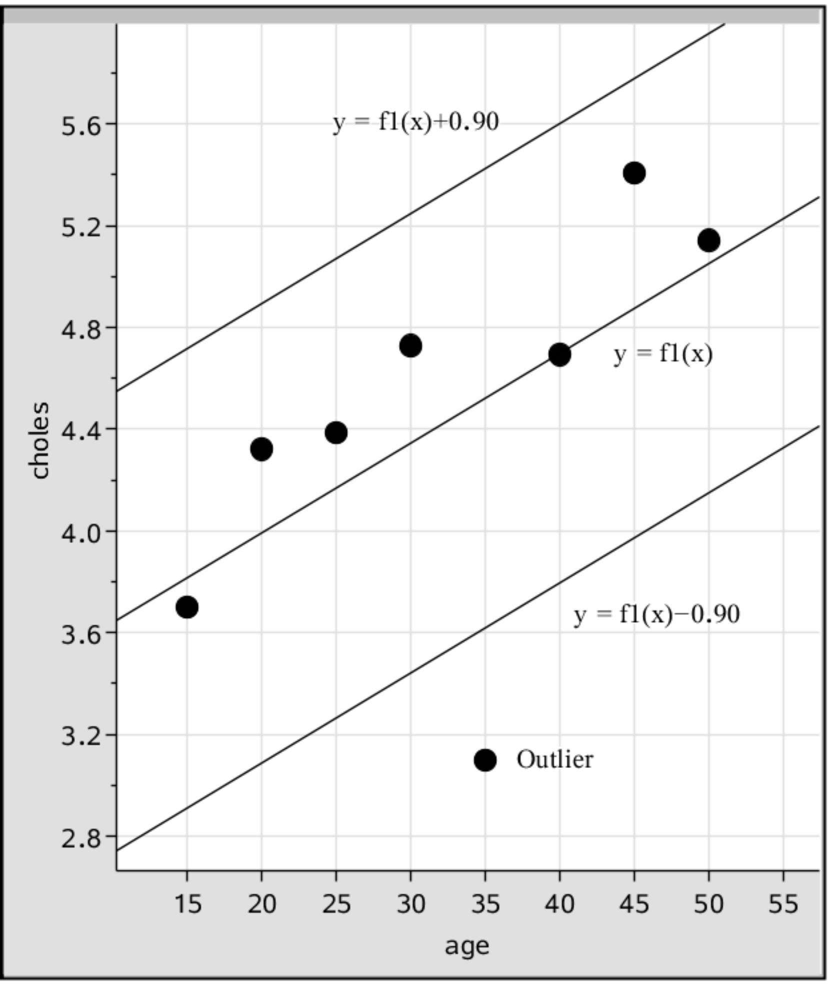
Correlation coefficient

stat.r = 0.575

c) On the diagram to the right you see the regression line $y = \text{f1}(x)$ and the lines $y = \text{f1}(x) \pm 0.90$ above and below the regression line.

There is only one outlier:

The observation at age 35.

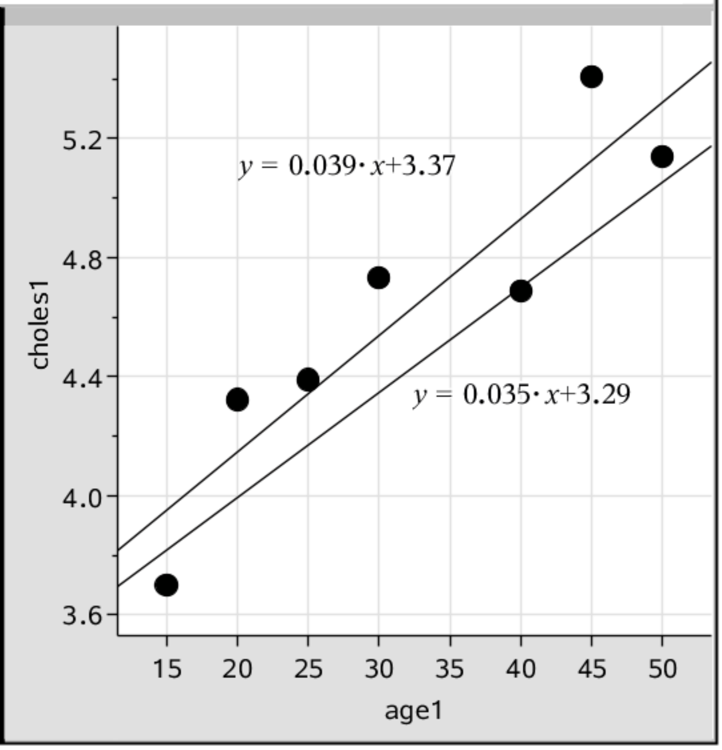


d) Below you see the spreadsheet without the outlier, the corresponding scatter plot and the two regression lines.

$y = \mathbf{f1}(x) = 0.035 \cdot x + 3.29$ and $y = \mathbf{f4}(x) = 0.039 \cdot x + 3.37$

	A age1	B choles1	C	D	E
=				=LinRegM	
1	15	3.7	Title	Linear R...	
2	20	4.32	RegE...	m*x+b	
3	25	4.39	m	0.039137	
4	30	4.73	b	3.36774	
5	40	4.69	r²	0.840457	
6	45	5.41	r	0.916764	
7	50	5.14	Resi...	{⁻0.2547...	
8					
9					

E8



d) cont.

The upper line (f4) is the new regression line.

Correlation coefficients: **stat.r** = 0.5754 for the first regression

and for the second regression **stat2.r** = 0.9168.

The correlation coefficient of the new regression is much closer to 1 that the first one.

As seen from the diagram the new regression line and the data points (without the outlier) show a good fit.

The first regression line gives higher y−values than the new one.

e) Cholesterol level at age 55 predicted by

first model: **f1**(55) = **5.23** and second model: **f4**(55) = **5.52** .

f)

solve(**f1**(x)=6,x) ▸ x=76.9

First model: Start medication just before age 77.

solve(**f4**(x)=6,x) ▸ x=67.3

Second model: Start medication just after age 67.