



S6MA3ENA
MATHEMATICS 3 PERIODS
B TEST
WITH CALCULATOR
DATE : 25/06/19
08 :30 - 10 :00
TEACHER : C. SEARLE

TiNSPIRE permitted in press-to-test mode.

/ 60	<i>Comments</i>	<i>Signature</i>
-------------	-----------------	------------------

TIME ALLOWED

- 90 minutes

NOTES

- Answer **all** questions.
- Answers must show reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of your answer.
- Unless indicated otherwise, full marks will not be awarded if the correct answer is not accompanied by supporting evidence of how the results have been achieved.
- When an answer provided is not the correct one, some marks can still be awarded if it is shown than an appropriate method and/or a correct approach has been used.

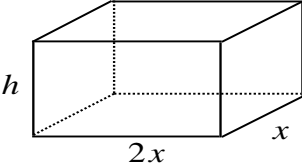
There are **6** questions on this paper.

Each question is worth **10 marks** with a total of **60 marks** available.

If you finish within the allocated time, read your answers and check that they are sensible.
Good luck !

<i>Functions</i>		
Q1	<p>The functions $f(x)$ and $g(x)$ are defined to be</p> $f(x) = x^2 + 2x \quad x \in \mathbb{R}$ $g(x) = \frac{1}{x+2} \quad x \in \mathbb{R}, x \neq -2.$ <p>(a) Draw a sketch of the graph of $f(x)$ labelling clearly the coordinates for the vertex and all points where the graph intersects the coordinate axes.</p> <p>The range for $g(x)$ is written $g(x) \in \mathbb{R}, g(x) \neq 0$.</p> <p>(b) What is the range for $f(x)$?</p> <p>(c) Write explicitly an expression for the composite function $g(f(x))$ and thus evaluate $g(f(2))$.</p> <p>(d) Solve the equation</p> $g(f(x)) = \frac{1}{10}.$ <p>(e) Is the function $g(f(x-1))$ an <i>odd</i> function or an <i>even</i> function? Give a reason for your answer.</p>	<p>[3 marks]</p> <p>[2 marks]</p> <p>[1 mark]</p> <p>[2 marks]</p> <p>[2 marks]</p>
<i>Arithmetic and Geometric Series</i>		
Q2	<p><i>You must justify your answers by writing all calculations that are relevant to your solutions.</i></p> <p>(a) The 4th term of a geometric series is 10 and the 7th term is 80.</p> <p>Use this information to find the common ratio and the first term for this series and hence find the sum of the first 15 terms.</p> <p>(b) A different series begins by adding the following numbers</p> $14 + 19 + 24 + \dots$ <p>How many terms of this series must be added together in order to exceed the sum of the first 15 terms of the geometric series?</p>	<p>[5 marks]</p> <p>[5 marks]</p>

	<i>Polynomial Differentiation</i>	
Q3	<p>The function $y = 2x^3 - 5x^2 - 4x + 2$</p> <p>is defined for $x \in \mathbb{R}$.</p> <p>(a) Use differentiation to determine the (x, y) coordinate for any stationary points of the function y</p> <p>(b) Classify the nature of any stationary points in terms of local maxima or minima.</p> <p>(c) Find the range of x values for which the curve is increasing.</p> <p>(d) Find the equation of the tangent line at $x = 1$</p>	<p>[2 marks]</p> <p>[3 marks]</p> <p>[2 marks]</p> <p>[3 marks]</p>
	<i>Probability</i>	
Q4	<p>Andy can walk to work, cycle or travel by bus. The choice he makes depends on the weather.</p> <p>(a) Copy and complete the tree diagram to show the probabilities for each of Andy's methods of travel.</p> <div style="text-align: center;"> <pre> graph LR Root(()) --- 0.8 Dry Root --- 0.2 Wet Dry --- 0.5 Walk1[Walk] Dry --- 0.1 Cycle1[Cycle] Dry --- 0.4 Bus1[Bus] Wet --- 0.1 Walk2[Walk] Wet --- 0.1 Cycle2[Cycle] Wet --- 0.8 Bus2[Bus] </pre> </div> <p>(b) A day is selected at random. Calculate the probability of the following events:</p>	<p>[2 marks]</p>

	<p>(i) The weather is wet and Andy travels by bus.</p> <p>(ii) Andy walks or cycles.</p> <p>(iii) The weather is dry given that Andy walks or cycles.</p>	<p>[2 marks]</p> <p>[3 marks]</p> <p>[3 marks]</p>
<i>Binomial Distribution</i>		
Q5	<p>On a certain remote island, 25 per cent of the population have the gene Hs which is known to protect against malaria.</p> <p>A random sample of 32 people have their blood tested to see if they have this gene.</p> <p>(a) Show that the sample will satisfy the conditions for a Bernoulli trial.</p> <p>(b) Determine the probability that the number that are found to have Hs is:</p> <p>(i) Exactly 5</p> <p>(ii) Fewer than 10</p> <p>(iii) At least 6 but at most 12</p> <p>(iv) More than the mean value for the distribution.</p> <p>(c) Will the conditions for a Bernoulli trial always be true for the population of the island? Write one or two sentences to justify your answer.</p>	<p>[2 marks]</p> <p>[1 marks]</p> <p>[1 marks]</p> <p>[2 marks]</p> <p>[2 marks]</p> <p>[2 marks]</p>
<i>Problem Solving with Maxima and Minima</i>		
Q6	<p>The length of the base of a cuboid is twice the width x, and its height is h centimetres, as shown in the diagram below.</p> <p>Its total surface area is $A \text{ cm}^2$ and its volume is $V \text{ cm}^3$.</p> <div style="text-align: center;">  <p>The diagram shows a 3D perspective of a cuboid. The front face is a rectangle with height labeled h and width labeled $2x$. The depth of the cuboid is labeled x. Hidden edges are shown with dashed lines.</p> </div> <p>(a) Show that $A = 4x^2 + 6xh$.</p> <p>The manufacturer needs the total surface area $A = 300$</p>	

	cm ² .	
(b)	Find an expression for h , in terms of x .	[2 marks]
(c)	Show that the volume $V(x) = 100x - \frac{4}{3}x^3$.	
(d)	Determine the maximum volume possible for the cuboid and determine the value of h that achieves this.	[2 marks]
	<i>You should explain in your answer how you know that this will be a maximum value.</i>	[1 mark]
		[5 marks]
<i>End of Examination</i>		