S6 MATHEMATICS – 3 Periods

PART B

DATE: 10th, June 2019

DURATION OF THE EXAMINATION: 90 minutes

Total: 65 points

With Calculator

SPECIFIC INSTRUCTIONS:

- Use a different page for each question.

- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.

- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.

- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.

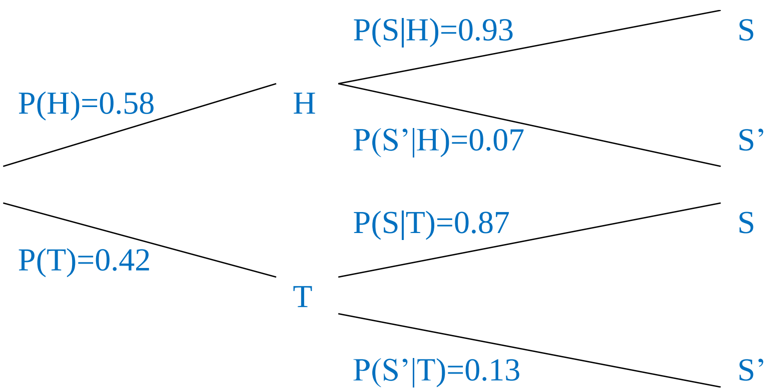
NUMBER OF PUPILS: 10

EXERCISE 1-B:

In New Portland there are only two car manufactures, Homba and Tayita. 58% of the cars are produced by Homba and the rest is produced by Tayita. No cars by other manufacturers exist.
7% of the cars produced by Homba do not meet the rigorous emission standards set by New Portland's environmental agency, whereas 87% of the cars produced by Tayita do meet these standards.

a. Draw a tree diagram illustrating this situation. All notation must be defined and all branches of the tree must be clearly labelled. **[5]**

Let's define the following events:
H: The car is produced by Homba
T : The car is produced by Tayita
S: The car meets the emission standards



b. What is the probability that a car meets the emission standards given that it was produced by Homba? **[2]**

$\boxed{P(S|H) = 0.93}$

c. What is the probability that a car was produced by Tayita **and** meets the emission standards? **[2]**

$$P(T \cap S) = P(T) \times P(S|T) = 0.42 \times 0.87$$
$$\boxed{P(T \cap S) \cong \ 0.37 = 37\%}$$

d. If a car is selected at random, what is the probability that it meets the emission standards? **[3]**

$$P(S) = \ P(H) \times P(S|H) + P(T) \times P(S|T) = 0.58 \times 0.93 + 0.42 \times 0.87$$

$0.58 \cdot 0.93 + 0.42 \cdot 0.87$ 0.9048

$$\boxed{P(S) \cong 0.90 = 90\%}$$

e. Given that a car meets the emission standards, what is the probability it was produced by Tayita?

$\dfrac{0.42 \cdot 0.87}{0.58 \cdot 0.93 + 0.42 \cdot 0.87}$ 0.403846

[3]

$$\boxed{P(T|S) = \frac{P(T \cap S)}{P(S)} \cong 0.40 = 40\ \%}$$

A study shows that the probability of a car meeting the emission standards is equal to 90%
Ten cars are randomly selected.

f. Write the formula that allows you to calculate the probability that k cars out of the ten chosen meet the emission standards. **The use of the formula must be carefully justified.** **[4]**

Let S represent the event the car meets the emission standards.

-We randomly select 10 cars therefore we have independent trials.
-Each trial has two possible outcomes (car meets standards/car does not meet standards) .
-Trials are repeated a finite number of times (n=10).
-The probability of success is constant $P(S) = p = 0.9$.

Let X be the number of cars meeting the emission standards among the 10 selected.
X follows a binomial distribution : $X \sim B(10, 0.9)$

$$P(X = k) = nCk \times p^k \times (1-p)^{n-k}$$

$$= 10Ck \times 0.9^k \times 0.1^{10-k}$$

g. What is the probability that all cars chosen will meet the emission standards? **[2]**

$$\textit{With the calculator}: \ P(X = 10) = binomPdf(10, 0.9, 10)$$

binomPdf(10,0.9,10) 0.348678

$$\boxed{P(X = 10) \cong \ 0.35 = 35\%}$$

h. What is the probability that 8 cars meet the emission standards? **[2]**

$$\textit{With the calculator}: \ P(X = 8) = binomPdf(10, 0.9, 8)$$

binomPdf(10,0.9,8) 0.19371

$$\boxed{P(X = 8) \cong \ 0.19 = 19\%}$$

i. What is the probability that at least 8 cars meet the emission standards? **[2]**

$$\textit{With the calculator}: P(X \geq 8) = binomCdf(10, 0.9, 8, 10)$$

binomCdf(10,0.9,8,10) 0.929809

$$\boxed{P(X \geq 8) \cong 0.93 = 93\%}$$

EXERCISE 2-B:

Professor Fry, a famous biologist, conducted a study on the population of viper snakes on an island of the coast of Brazil known as Snake Island.

When the study began, the population of this endangered species was 4000 individuals. The study indicated that the population was **decreasing** by 5% each year due to competition for resources.

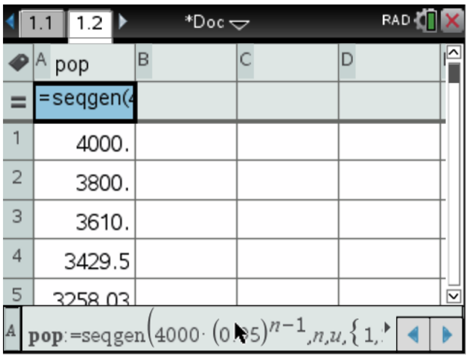
a. Write a formula for the population in year n (u_n). Justify. **[3]**

The number of individuals at the beginning of each year is given by a Geometric sequence

with $u_1 = 4000$ and $r = 0.95$ $(decrease\ of\ 5\%)$

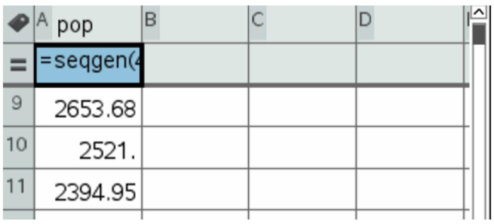
$$\boxed{u_n = 4000 \times 0.95^{(n-1)}}$$

b. **Copy and complete** the table **[1.5]**



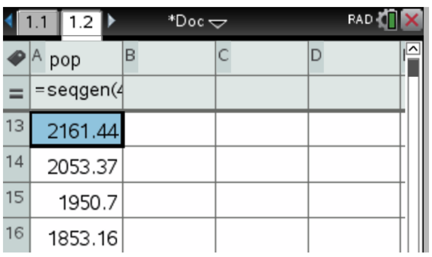
Beginning of year	1	2	3	4
Population	4000	3800	3610	3429

c. What will the population be at the beginning of year 10? **[1.5]**



$\boxed{At\ beginning\ of\ year\ 10\ the\ population\ will\ be\ 2521\ snakes.}$

d. When was the initial population halved? **[2]**



Half of the initial population is 2000 snakes.

$\boxed{The\ initial\ population\ was\ halved\ during\ the\ 14th\ year}$

After 15 years the trend was reversed and the population started increasing following the formula

$$P(n) = 500 + \frac{4000}{2+(0.7)^n}$$ (n is the number of years from year 15 onwards)

e. Due to the limited amount of resources, the island can only sustain the life of 2800 individuals. Is this population growth sustainable? **Justify your answer**. **[2]**

1.1 1.2 *Doc RAD

	A pop	B newpop	C	D
=	=seqgen(	=seqgen(		
37		2500.		
38		2500.		
39		2500.		
40		2500.		

B **newpop**:=seqgen$\left(500+\frac{4000}{2+(0.7)^n},n,u,\{\right.$

The population eventually stabilizes at 2500 snakes. That number is lower than 2800, the number of snakes that can live on the island.

$\boxed{\textit{Therefore this growth is sustainable.}}$

EXERCISE 3-B:

Australian biologist, Professor Fry, conducted a study on the length of the Golden Lancehead viper, a venomous snake, living on Snake Island in Brazil.
He measured the length size of a sample of snakes and recorded his data into the following table.
(Length values in table were rounded to the nearest cm)

Length (cm)	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89
Frequency	20	31	52	56	65	68	72	90	80	82	38	19	12	9	6

	A length	B freq	C cumfreq
=			
1	75.	20.	20.
2	76.	31.	51.
3	77.	52.	103.
4	78.	56.	159.
5	79.	65.	224.
6	80.	68.	292.
7	81.	72.	364.
8	82.	90.	454.
9	83.	80.	534.
10	84.	82.	616.
11	85.	38.	654.
12	86.	19.	673.
13	87.	12.	685.
14	88.	9.	694.
15	89.	6.	700.

D	E
	=OneVar(
$\bar{x}$	81.11
Σx	56777.
Σx²	4.61182...
sx := sn-...	3.08175
σx := σn...	3.07955
n	700.
MinX	75.
Q₁X	79.
MedianX...	81.
Q₃X	83.
MaxX	89.
SSX := Σ..	6638.53

700 values

Position of Median: Between 350th and 351th

Median= 81 cm

Position of Q1: Between 175th and 176th

Q1= 79 cm

Position of Q3: Between 525th and 526th

Q3= 83 cm

	A length	B freq	C	D
=				=OneVar(
2	76.	31.	$\bar{x}$	81.11
3	77.	52.	Σx	56777.
4	78.	56.	Σx²	4.61182...
5	79.	65.	sx := Sn-...	3.08175
6	80.	68.	σx := σn...	3.07955

D2 =81.11

	A length	B freq	C	D
=				=OneVar(
8	82.	90.	MinX	75.
9	83.	80.	Q_1X	79.
10	84.	82.	MedianX...	81.
11	85.	38.	Q_3X	83.
12	86.	19.	MaxX	89.

D2 =81.11

a. Calculate the mean, median, mode and range for this data set. **[2]**

$$\boxed{Mean = 81.11\ cm \quad Median = 81\ cm \quad Mode = 82\ cm \quad Range = 89 - 75 = 14cm}$$

b. Calculate Q1 and Q3 for this data set. **[2]**

$$\boxed{Q1 = 79\ cm \qquad Q3 = 83\ cm}$$

c. Calculate the standard deviation for this data set. **[1]**

$$\boxed{\sigma \cong 3.08\ cm}$$

d. Give an interpretation of the standard deviation. **[2]**

The standard deviation is how far in average the data values are from the mean.

e. What percentage of the snakes measured 80 cm? **[2]**

$$\boxed{\frac{68}{700} \cong 0.10 = 10\%\ of the\ snakes\ measured\ 80\ cm}$$

f. What percentage of the snakes measured less than 80 cm? **[2]**

	A length	B freq	C cumfreq	D
=				
1	75.	20.	20.	Title
2	76.	31.	51.	$\bar{x}$
3	77.	52.	103.	Σx
4	78.	56.	159.	Σx²
5	79.	65.	224.	sx := Sn-...

C5 =c4+b5

There are 224 snakes measuring less than 80 cm.

$$\boxed{\frac{224}{700} \cong 0.32 = 32\%\ of\ the\ snakes\ measure\ less\ than\ 80\ cm}$$

EXERCISE 4-B:

Professor Fry invested a sum of money in a bank account at the start of the year. The bank gives him the same interest rate every year. At the start of the 5th year, his investment will be worth 236 150 € and at the start of the 10th year his investment will be worth 287 313 €.

a. Find the interest rate he gets. **[5]**

The sum of money at the beginning of each year is given by a geometrical sequence.

$u_5 = 236\,150$, $u_{10} = 287\,313$ *and* $u_{10} = u_5 \times r^5$

```
solve(287313=236150·r^5,r)        r=1.04
```

$\boxed{r = 1.04 \text{ therefore the interest rate is } 4\%}$

b. Find the original amount of money he put into the account. **[5]**

$u_5 = u_1 \times r^4$

```
solve(236150=u1·(1.04)^4,u1)        u1=201862.
```

$\boxed{\text{The original amount of money he put into the bank account was } 201\,862\ €}$

EXERCISE 5-B:

Professor Fry and 11 colleagues from his team went to a restaurant to commemorate the success of the research conducted on Snake Island. At the end of the meal, they decided to randomly select three members of the group to pay the bill.

a. In how many ways can these three people be selected? **[3]**

Here the order is not important. We select 3 people among 12 → 12C3

```
nCr(12,3)        220.
```

$\boxed{\text{Three members of the group can be selected in } 220 \text{ different ways.}}$

b. What is the probability that Professor Fry will **not** have to contribute to the payment? **[3]**

```
nCr(11,3)/nCr(12,3)        0.75
```

$$\boxed{P(\text{Fry is not selected}) = \frac{11C3}{12C3} = 0.75 = 75\%}$$

c. In the group there are 3 biologists, 5 zoologists and 4 veterinarians. What is the probability that exactly one of each profession will be selected? **[3]**

```
nCr(3,1)·nCr(5,1)·nCr(4,1)/nCr(12,3)        0.272727
```

$$\boxed{P(1B \text{ and } 1Z \text{ and } 1V) = \frac{3C1 \times 5C1 \times 4C1}{12C3} \cong 0.27 = 27\%}$$