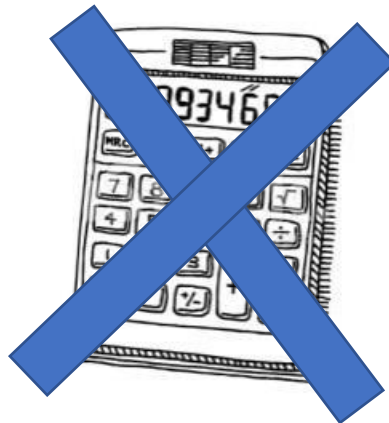# MATHEMATICS 3
Part A

Date: Wednesday 15th December 2021

DURATION OF EXAMINATION:
1 hour 30 minutes (90 minutes)
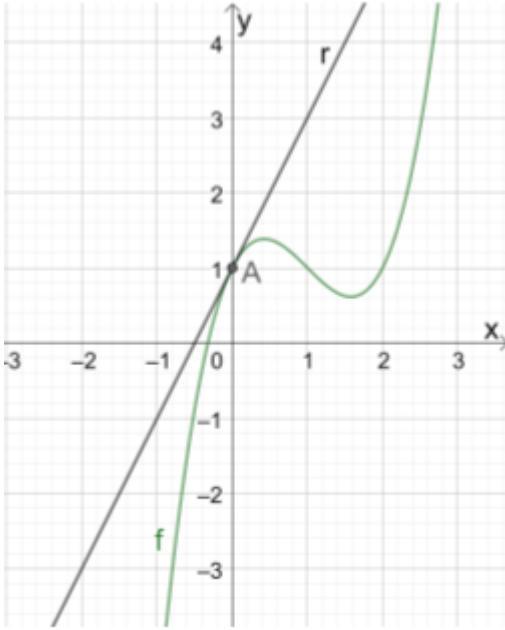
Answer ALL questions

SPECIFIC INSTRUCTIONS:

- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.

- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.

- When the answer provided is not the correct one, some marks can be awarded if it is evident that an appropriate method and/or a correct approach has been used.

K. Osborne

| PART A	Marks
1) Give the derivative $f'(x)$ of the following functions:	
(a) $f(x) = x^3 - 3x^2$	2
(b) $f(x) = 2x^2 + x - 3$	2
(c) $f(x) = \frac{1}{2}x - \frac{1}{3}x^3 + \frac{2}{3}x^6$	2
2) Consider the graph of the function f shown below. The line r is a tangent line to the graph of f at point A. 	
(a) Use the information in the diagram to find the equation of the line r.	4
(b) Given that $f(x) = x^3 - 3x^2 + 2x + 1$, use the diagram or otherwise to find the value of $f'(0)$.	4

3) A town`s population is growing linearly. In 2018 the population was 5000. By 2020 the population had increased to 7400.

(a) Give the function $P(t)$ where P is the population and t is the number of years since 2018.

3

(b) Use your function $P(t)$ to predict the population in 2025.

2

(c) According to this model in which year will the population reach 19400?

2

4) The function f is defined as $f(x) = 2x^2 - 8x + 8$.

(a) Determine the coordinates of the y-intercept.

2

(b) Calculate $f(2)$

2

(c) Determine the derivative $f'(x)$.

2

(d) For what value of x does the function $f(x)$ have a turning point? State the nature of the turning point and explain your answer.

3

(e) Find the equation of the tangent to the curve at the point (1,2).

4

(f) The point A is a point on the graph of f. The gradient at the point A is equal to 12. Find the coordinates of the point A.

4

5) The diagram below shows the graph of the function $f(x) = \frac{ax+b}{x+c}$.
The dotted blue lines represent the asymptotes. The graph passes through the point $\left(0, -\frac{1}{2}\right)$.

(a) Give the equation of the vertical asymptote.   2

(b) State the domain of the function.   2

(c) Find value of c.   2

(d) Give the equation of the horizontal asymptote.   2

(e) State the range of the function.   2

(f) Find value of a.   2

(g) A student says that the value of b is 1. Are they correct? You must justify your answer.   2

4
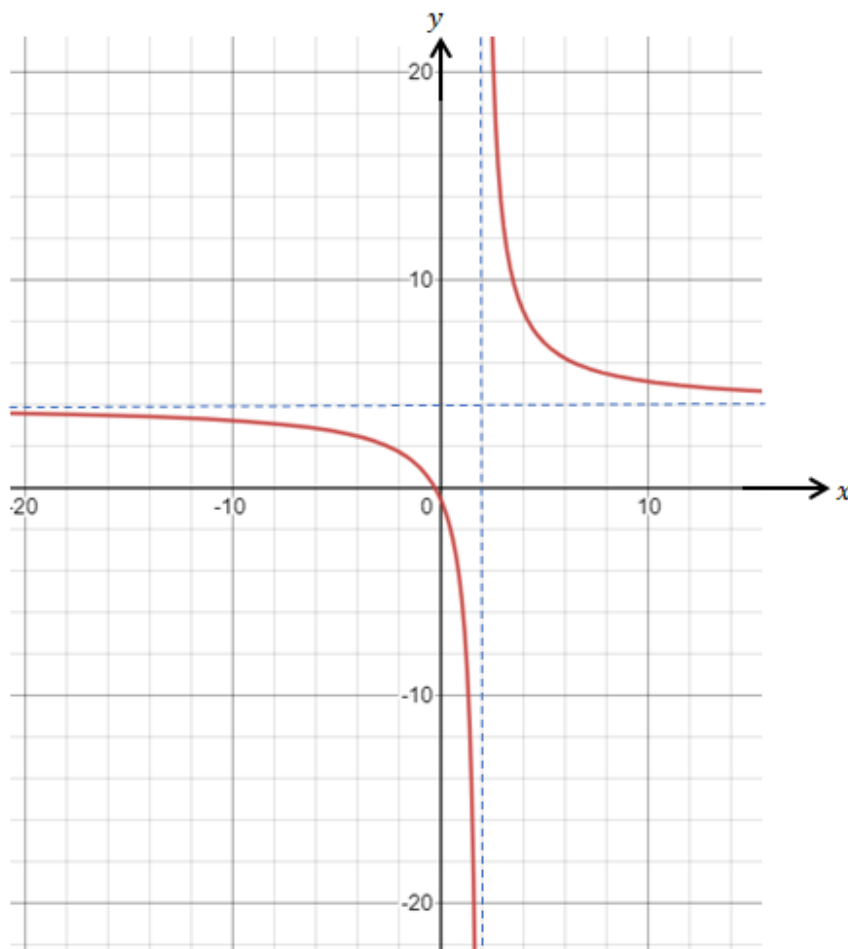
6) The graph of the derivative $f'(x)$ is given below.
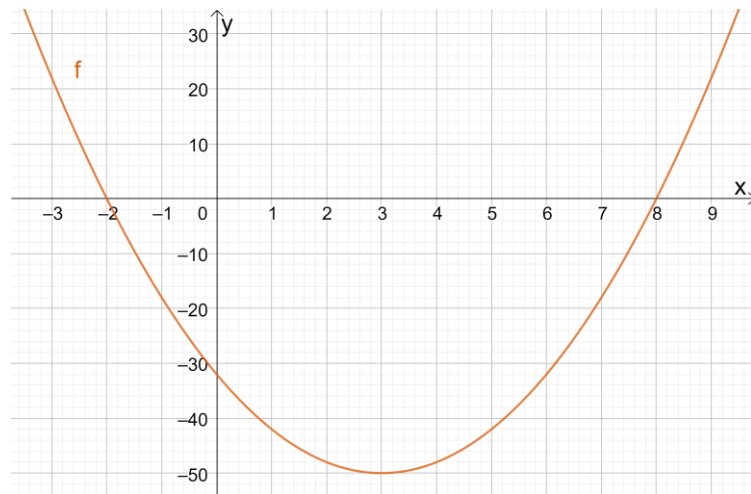
a) Give the x-coordinates of the two turning points. 2

b) For which values of x is the graph of $f(x)$ increasing? 2

c) For which value of x does $f(x)$ reach a minimum? 2

d) Sketch a possible graph of $f(x)$, given that the point $(8,0)$ lies on 3
    the graph of $f(x)$.

7) The graph below is the graph of the function $f(x)$.
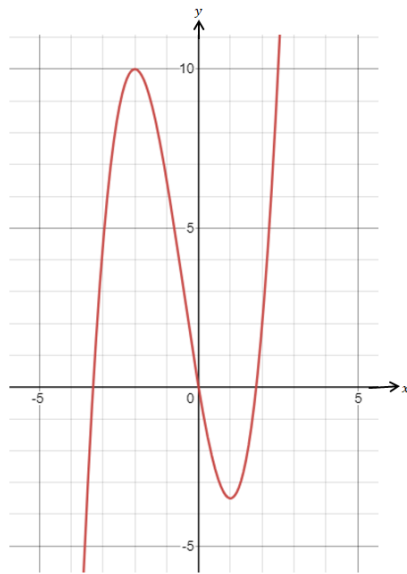
Which of the 4 graphs below is the corresponding graph of $f'(x)$?

4

For each graph you **must** explain why it is or is not the correct graph.

A

B

C

D