PreBAC

Maths 3

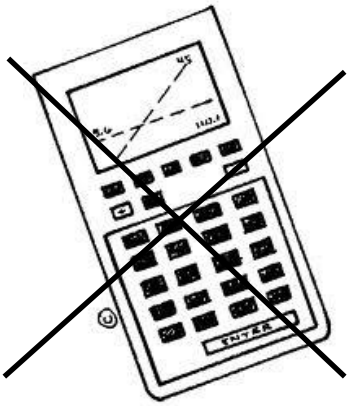
ENGLISH Solutions

DATE: January 30, 2023

Total : ..... /50 points

DURATION OF THE EXAMINATION:

2 hours (120 minutes)

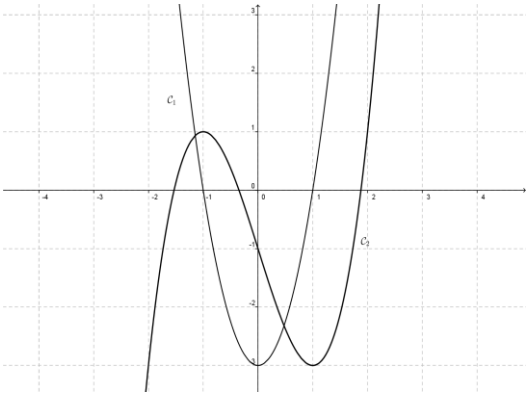


EQUIPMENT AUTHORIZES:

Exam without technological support

SPECIFIC INSTRUCTIONS:

- It is essential that the answers be accompanied by the explanations necessary for their preparation.
- Responses should highlight the reasoning that leads to the results or solutions.
- When graphs are used to find a solution, the response should include sketches of them.
- Unless otherwise stated in the question, all points cannot be attributed to a correct answer in the absence of the reasoning and explanations that make it possible to arrive at the results or solutions.
- Where an answer is incorrect, however, part of the points may be awarded when an appropriate method and/or correct approach has been used.

1)	A function is defined by $f(x) = k \cdot x^2 - 2x$ with $k \in \mathbb{R}., any\ real\ number$ **Calculate** k , so that $f'(1) = 4$ Solution : $f'(x) = 2.k \cdot x - 2$ $f'(1) = 4 \quad therefore\ 2.k \cdot (1) - 2 = 4$ $2.k = 6$ $k = 3$	**5 points**
2)	The curves in the diagram are graphs of a function and its derivative. Clearly determine and justify which curve is the function and which is its derivative. $C_1\ C_2$  Solution: c1 is a quadratic and c2 is a cubic; The cubic C2 has a higher order and so is the function; and C1 is the derivative. However we must check these 2 are actually linked to each other The turning points, max/min of C2 have the same x values as the intercepts of c1. The gradient at the turning points is zero so the derivative function should have zeros at this point: hence it is the dervative The function c2 has a positve slope x<-1 and x>1 it has a negative slope from -1<x<1 This matches when c1 is positive and nagative	**5 points**

3)	A function is defined $$f(x) = \frac{1}{3}x^3 - 3x^2 - 5x + 6$$ **Calculate** the tangent equation for this function when $x = 0$ Solution : $y = mx + c \quad or \quad f(x) = f'(x)x$ $f(0) = \frac{1}{3}(0)^3 - 3(0)^2 - 5(0) + 6$ $f(0) = 6 \quad y = 6$ $f'(x) = \frac{1}{3}.3x^2 - 3.2x - 5 \qquad f'(x) = x^2 - 6x - 5$ $f'(0) = (0)^2 - 6(0) - 5$ $f'(0) = -5 \qquad$ or $m = -5$ $y = mx + c \quad or \quad f(x) = f'(x)x$ $6 = -5(0) + c \qquad c = 6$ $Tangent\ line\ is\ given\ by: \qquad y = -5x + 6$	**5 points**

4)

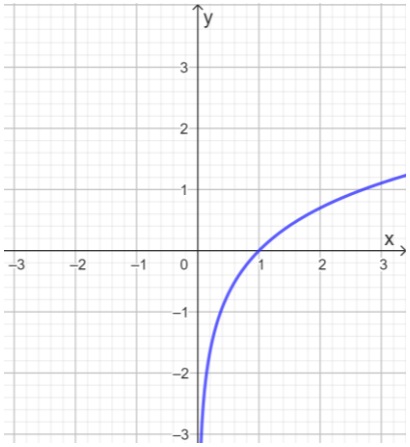
Match each of the equations (A, B, C , D and E)

with their corresponding graphs (1,2,3,4 and 5),

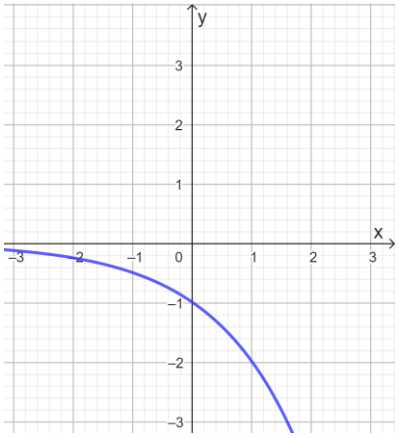
A:$y = 2^{2x}$ B : $y = -2^{x}$ C:$y = \ln x$

D:$y = 2^{-x+1}$ E:$y = \ln(x+2)$

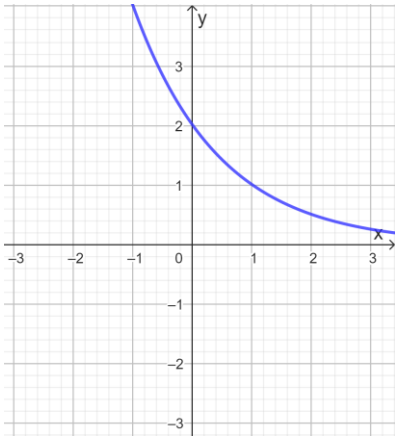
1.)



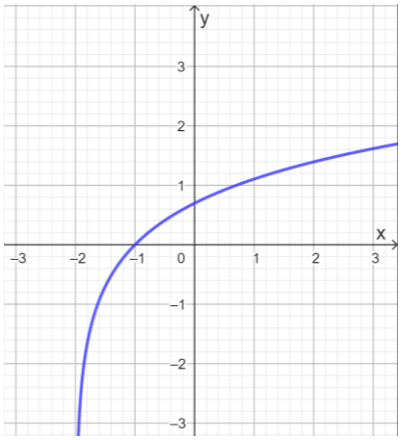
2.)



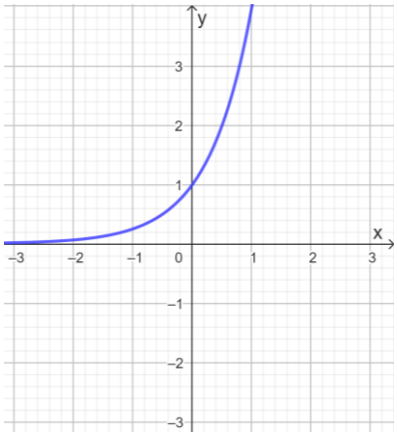
3.)



4.)



5.)



Solutions:

A = 5

B= 2

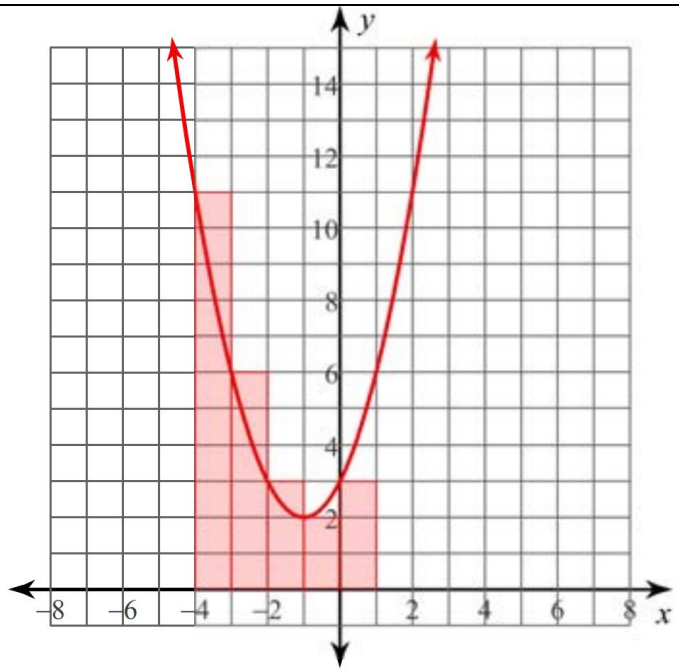
C= 1

D= 3

E = 4

5 points

5) **5 points**



The Graph shows a curve of the function defined by:

$$f(x) = x^2 + 2x + 3$$

A student wants to find an approximation of:

$$\int_{-4}^{1} f(x)\,dx$$

a) **Explain,** referring to the graph, what this notation means.

b) Using the graph, **estimate** this value by calculating pink rectangles.

c) Do you think this estimate is above or below the real value? Explain

Solution :

a) The symbol means an integral and to find the value of this integral between the values of $-4 < x < 1$, this is represented by the total area under the curve bounded by the lines x=-4 and x=1

b) Adding the rectangles (11x1)+(6x1)+(3x1)+(2x1)+(3x1)
Integral has the value 25

c) This estimate is an over estimate as the area of rectangles above the line of the function is greater than the gaps not calculated under the curve. Approx 2 square units below not added and maybe as much as 4 squares over the line.

6)	Marc and Jeff play 4 tennis matches against each other. The probability of Marc winning a match is $\frac{1}{3}$. The results of each match are independent. Calculate the probability that Marc will win exactly one of the 4 matches. Solution Probability of Marc winning is $\frac{1}{3}$ and Marc loosing is $\frac{2}{3}$ X is the probability of Marc winning Using the formula $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$ n=4 k =1 $P(X=1)=\frac{4!}{3!\,1!}.\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$ $P(X=1)=4\left(\frac{1}{3}\right)\left(\frac{8}{27}\right)$ P that Marc wins exactly one match is $\frac{32}{81}$	**5 points**

7) **5 points**

A survey of 80 S7 students about their choice of subject options showed that:

- 20 chose physics.

- 33 chose economics.

- 41 chose neither physics nor economics.

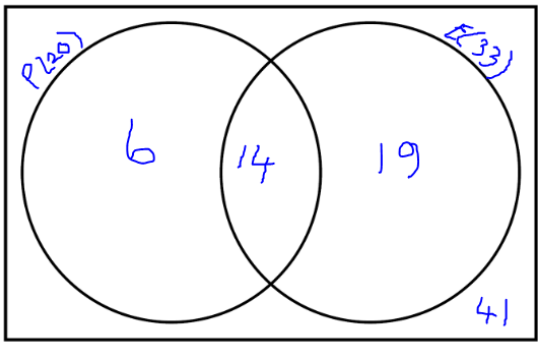
a) Plot the results of this survey using a Venn diagram or a double-entry table.

b) How many students chose physics or economics?

(c) A student is interviewed at random. Knowing that he chose physics, what is the probability that he also chose economics?

Solution :P = Physics students E= Economics students

U= 80



Subject Studied	Economics	Not Economics	Totals
Physics	14	6	20
Not Physics	19	41	60
	33	47	80

B) 80-41 = 39 So 39 Students choose physics or Economics

(should not be 25 as includes students who do both E and P)

C) 20 students chose physics and of those 14 do Economics

Therefore Probability of doing Economics; given that they student is taking physics is $\frac{14}{20}$ or $\frac{7}{10}$ 0.7 or 70%

8)

During the International Kite Festival, a stand organizes a contest to win a kite.

There are 10 cards face down on the table, 7 are red and 3 are black.

There is a six-sided die to roll.

You win if you choose a red card and throw a 5.

or if you choose a black card and throw an even number.

a) Justify, by calculations , with which of these two possibilities one has the greatest probability of winning.

b) Determine the total probability of winning the kite

Solution :

Probability of choosing a red card is $\frac{7}{10}$

Probability of rolling a 5 is $\frac{1}{6}$

Probability of choosing a red and rolling a 5 is $\frac{7}{10}.\frac{1}{6}=\frac{7}{60}$

Probability of choosing a black is $\frac{3}{10}$

Probability of rolling an even number is $\frac{3}{6}$ or $\frac{1}{2}$

Probability of choosing black and rolling even is $\frac{9}{60}$

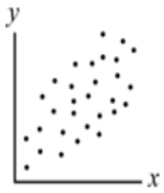
$\frac{9}{60}>\frac{7}{60}$ So black card and even die number is more likely than Red card and 5 on the die

B) The probability of winning is sum of 2 probabilities calculated above:

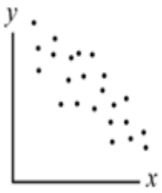
$\frac{9}{60}+\frac{7}{60}=\frac{16}{60}$ or $\frac{4}{15}$

5 points

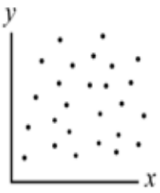
9)



A



B



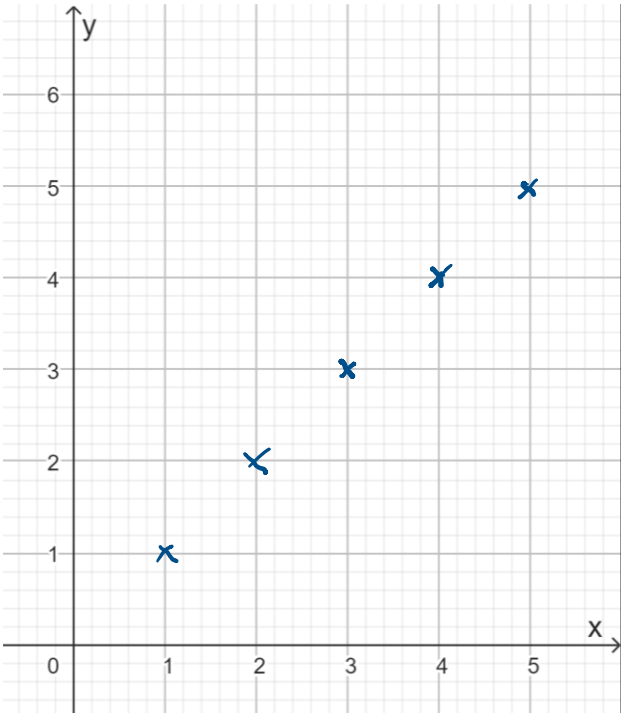
C

D

i) Explain for which figure(s), from A B C and D, it would be appropriate to model the situation as a linear correlation.

ii) Explain whether scatterplot B has a positive or negative correlation coefficient r.

iii) **Copy** the illustrated coordinate system below and draw a simple scatterplot (at least 5 points) such that the relationship would be considered a linear correlation with the correlation coefficient $r \approx 1$.



Solutions

i) A and B can be modelled appropriately by linear ;
C has no real correlation and D is not linear – a curve

ii) B is negative as the slope of the line through the points would have a negative gradient, it is decreasing

iii) Any 5 points where you can almost perfectly draw a straight line through the points

A best fit line was not asked for just a scatter plot.

5 points

10)

The number of Ladybirds N(t) living on a rose bush in June is given by the model:

$$N(t) = 6.e^{\ln(1.16).t}$$

Where t is the number of days, and t = 0 is on June 1st.

a) How many ladybirds were there on the bush on June 1st?

b) Rewrite the equation as:

$$N(t) = a.b^t$$

The number of greenfly G(t) on the same rose bush is modelled by the following equation:

$$G(t)=1500.(0.68^t)$$

c) Is the greenfly population increasing or decreasing?

d) Give this change as a percentage change per day.

5 points

Solution

a.) On June first t= 0 therefore $N(0) = 6.e^{\ln(1.16).0}$

As $e^{.0} = 1$ N(0)=6

b.) $N(t) =(6).(1.16)^t$ $N(t) =(6) \times (1.16)^t$

c.) The population of Greenfly is decreasing as the b value is less than 1

d.) 0.68 represents a decrease of 32% in the number of greenfly per day on the rose bush.
(1-0.68 = 0.32)