MATHEMATICS 3 PERIODS

PART B

ENGLISH version

DATE: Monday 30th January 2023,

Total :………… / 50 points

EXAM DURATION: 2 hours (120 minutes)

AUTHORISED EQUIPMENT:

Exam with technological support:

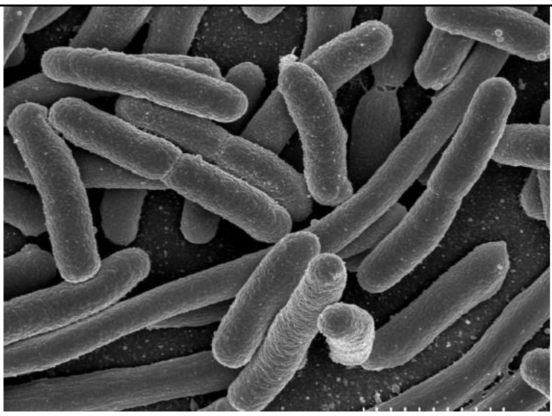
Calculator allowed

Pencil for graphics

SPECIAL NOTES:

- It is essential that the answers be accompanied by the explanations necessary for their preparation.
- Responses should highlight the reasoning that leads to the results or solutions.
- When graphs are used to find a solution, the answer should include sketches of them.
- Unless otherwise stated in the question, all points cannot be attributed to a correct answer in the absence of the reasoning and explanations that lead to the results or solutions.
- Where an answer is incorrect, however, part of the points may be awarded when an appropriate method and/or correct approach has been used.

PART B Question 1 : 22 points	Points



a) E. coli bacteria are usually found in the lower intestines of humans and other warm-blooded organisms. They reproduce at a rate of 3.5% per minute.
Scientists observe a colony of 100 000 bacteria at the beginning of the experiment.

Write the equations which would model the growth of E. coli bacteria in the form:

$$f(t) = a \cdot b^t$$

represents $f(t)$ the number of bacteria at a certain time t and t represents time in minutes.

Solution

$$f(t) = 100\,000 \cdot 1.035^t$$

/2

b) Bifidobacterium is the most common bacterium in the gut microbiome of infants. Some bifidobacteria are used as probiotics. We know from previous studies that a colony of bifidobacteria grows in the following pattern:

$$g(t) = 200\,000 \cdot 1{,}05^t$$

$g(t)$ represents the number of bacteria at a certain time t
t represents time in minutes.

i. Calculate the number of bifidobacteria when t = 0 and t = 30 to the nearest unit.

$g(0) = 200\,000 \cdot 1.05^0$ = 200 000 There are 200 000 bacteria at the start

/4

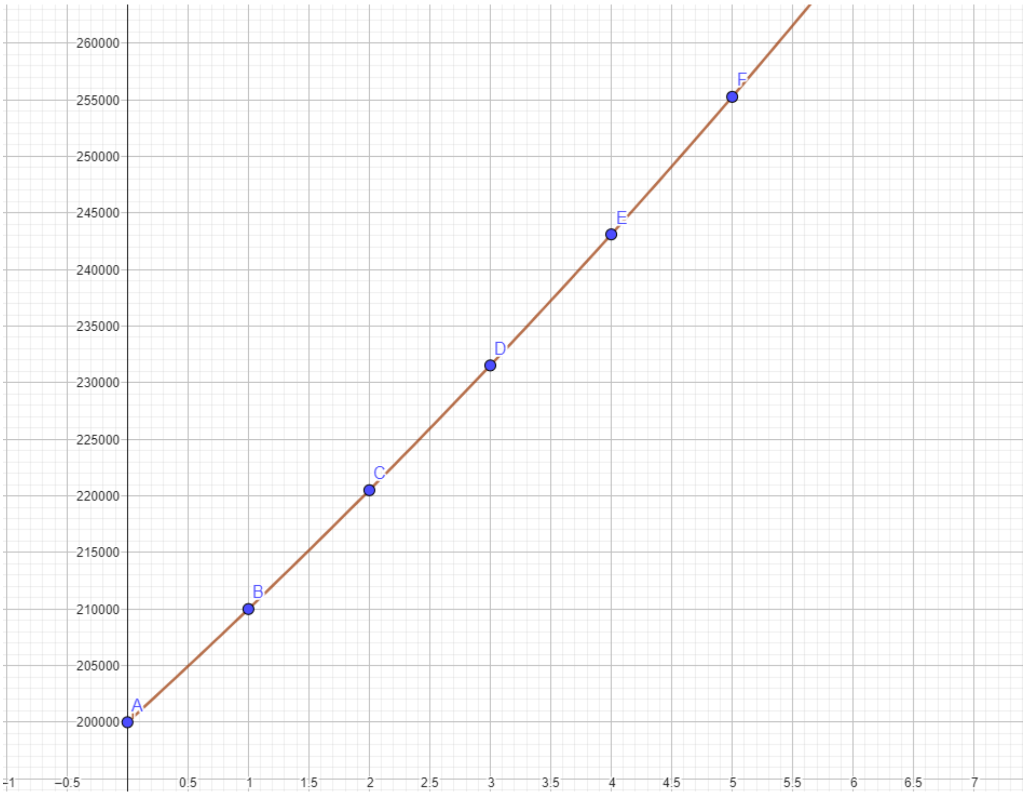
$g(30) = 200\,000 \cdot 1.05^{30}$=864 388 There are 864 388 bacteria after 30 minutes

(no unit - 1)

ii. Copy and complete this table and plot graph g for 0≤ t ≤5 in an appropriate coordinate system.

t	0	1	2	3	4	5
$g(t)$	200 000	210 000	220 500	231 525	243 101	255 256

/4



Axis labels ; y = number of bacteria; x = Time in minutes

iii. The experiment's nutrient solution can only accommodate 10 million bacteria. Calculate when the colony reaches this number. The answer should be rounded to the nearest minute.

Solve $g(t) = 200\ 000 * 1.05^t$

Via logs

$10\ 000\ 000 = 200\ 000 \cdot 1.05^t$

$\frac{10\ 000\ 000}{200\ 000} = 1{,}05^t$

$50 = 1{,}05^t$

$log50 = log1.05^t$ $\qquad$ $log50 = tlog1.05$

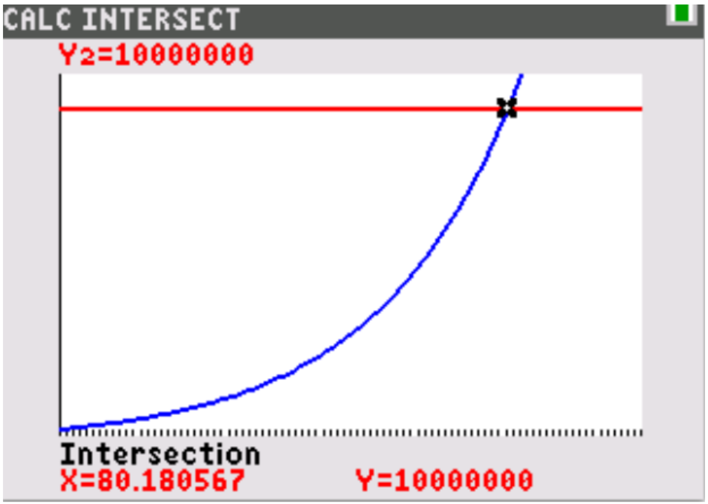
t = $\frac{log50}{log1.05}$ = 80.18

If logs rounded = 1.699/0.021 = 80.905 ie 81 days

2 table, 2 graph

Graphically looking at intersection of y=10 000 000

/2



The population of bacteria reaches 10 000 000 after 80 minutes

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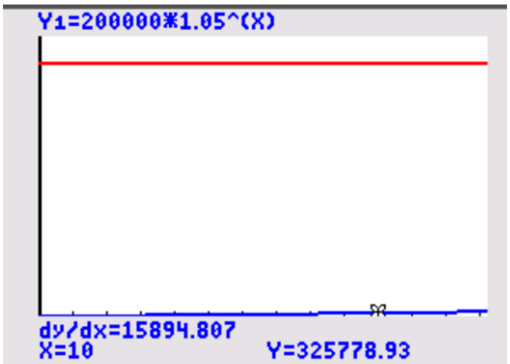
iv. Calculate the growth rate g'(10) rounded to the integer and interpret the result in the context of the exercise.



After 10 minutes the bacteria population is growing at a rate of 15 895 bacteria per minute /2

c)

Bacterial disease of tomato fruit spots is caused by the bacterium Xanthomonas vesicatoria. The infection causes brown spots on leaves and fruits and can lead to significant yield losses. /2 We know from experience that the probability of a tomato plant being infected with the bacteria is 2.5%.

A farmer owns a small field with 500 tomato plants.

i. Indicate how many infected plants are expected to be found.

/3

Solution 500 x 0.025 = 12.5
We would expect to find 13 infected plants.

ii. Calculate the probability that only 2% of tomato plants will be infected.

2% of plants is 500x0.02 =10

Binomial Pdf n= 500, p = 0.025 X = 10

$$P(X = 10) = \binom{500}{10}(0.025)^{10}(1 - 0.025)^{490}$$

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binompdf(500,0.025,10)
                 0.09599804
```

iii. Calculate the probability that between 10 and 20 plants (including both numbers) will be infected.

Proabability of between 10 and 20
Binomial Cdf X=20 – Binomial Cdf X=9

9	0.198
10	0.294
11	0.4037
12	0.5183
13	0.6285
14	0.7269
15	0.8086
16	0.8721
17	0.9185
18	0.9504
19	0.9711
20	0.9839

$P(10 \le X \le 20) = 0.9839 - 0.198$
$= 0.7859$

Or method 2

Binomial Pdf add values of 10 through 20 = 0.786

X	Y1
10	0.096
11	0.1096
12	0.1146
13	0.1103
14	0.0984
15	0.0817
16	0.0635
17	0.0464
18	0.0319
19	0.0208
20	0.0128

PART B Question 2 : 28 points	Points

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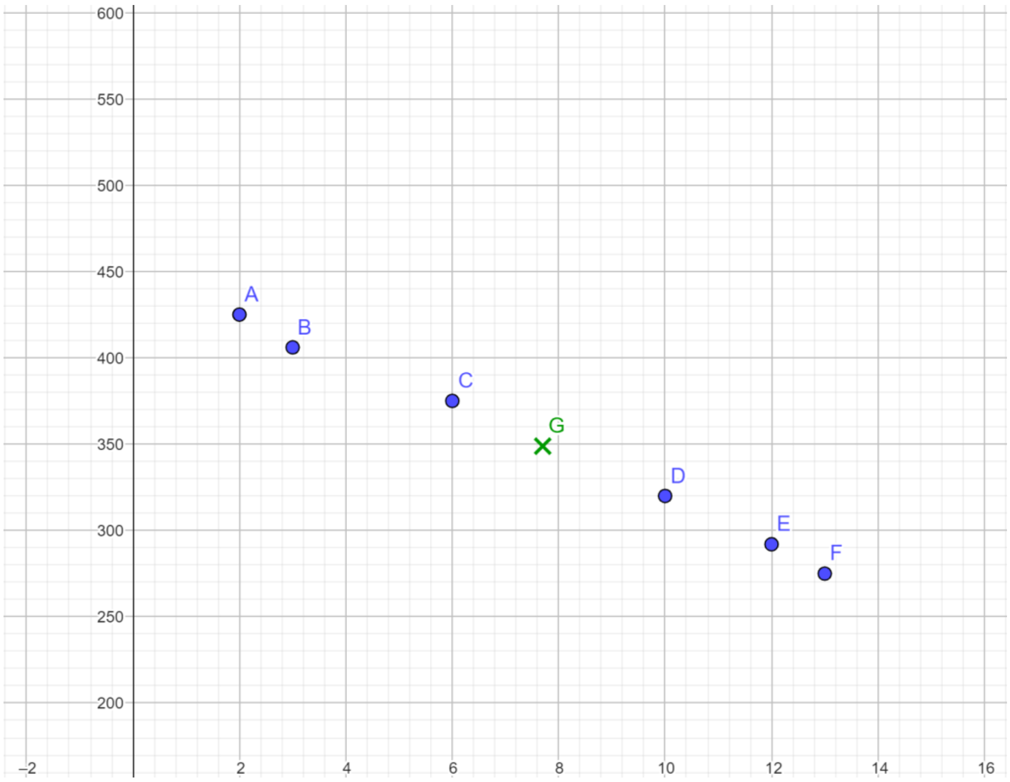
The Town of Mickey-Town evaluates speeding data on local roads in relation to the number of educational radar signs installed. The following table shows the number of signs installed and speeding fines over the past six years:

number of educational radar signs (x) 	2	3	6	10	12	13
number of speeding fines (y)	425	406	375	320	292	275

a) Plot the table data in a scatterplot:
(Suggested scale on the x axis use 1 cm for a sign and on the y axis 1 cm for 20 fines starting the scale from 180).



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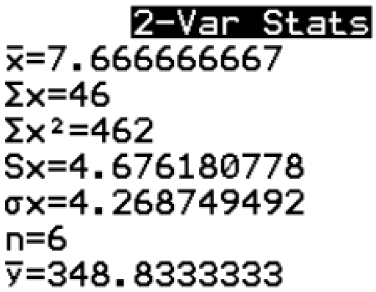
b) Calculate the mean value $\bar{x}$ of the number of signs over the six years.
Round to one decimal place



/2

Average number of signs x = 7.7 over the 6 year period

/2

(c) Calculate the mean value $\bar{y}$ of the number of fines. Round to one decimal place

Average number of fine Y over the 6 years is 348.8

/1

d) Draw the mean point ($\bar{x}$, $\bar{y}$) onto the graph and label it. (see graph above pt G)

(e) Give the values of σx and σy Round to one decimal place

/2

Standard deviation of x is 4.3 and of y is 56.7

```
2-Var Stats
↑σx=4.268749492
 n=6
 ȳ=348.8333333
 Σy=2093
 Σy²=749375
 Sy=62.07549167
 σy=56.66691176
```

f) Calculate the linear correlation coefficient and explain whether a linear model is appropriate or not. /2

```
LinReg
y=ax+b
a=-13.25609756
b=450.4634146
r²=0.9971801662
r=-0.9985890878
```

r= -0.9985 which suggests a very strong negative linear correlation, so yes the linear model is appropriate

g) Determine the equation of the line corresponding to the linear correlation fit. Using the least squares regression method. /2

In the form $y = ax + b$
Round the values of a and b to 2 decimal places.

y= -13.26 +450.46

h) The company used the following linear regression equation $y = -13x + 450$,
Use this to estimate the number of fines if there were 15 signs. /2

y = -13(15) +450
y= -195+450
= 255

(i) The profit of the educational radar sign company is represented by the function:

$$B(x) = \frac{x^3}{3} - 16x^2 + 220x\,, \quad 0 \le x \le 18$$

Where x is given in hundreds of radar signs produced. B is given in euros

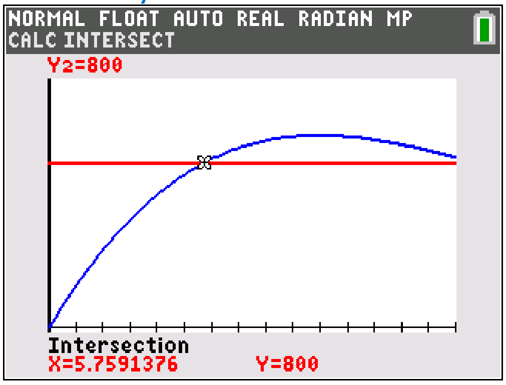
i. What is the profit for selling 900 radar signs? /2
If 900 radar x is 9 because (900 /100 = 9)

$$B(9) = \frac{(9)^3}{3} - 16(9)^2 + 220(9) = 927\ Euro$$

If they do B(900) correct 1 mark - 230 238 000 Euro

ii. How many radar signs does the company have to sell to make a profit of 800 €?

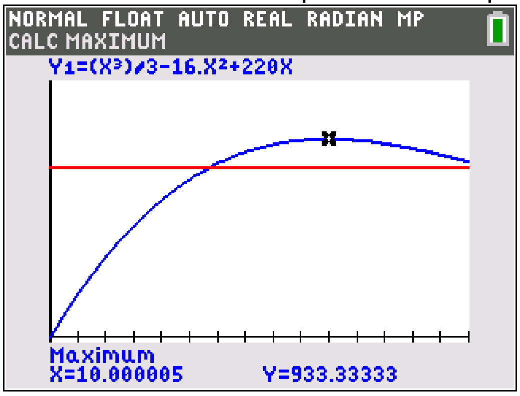
Solve $800 = \frac{x^3}{3} - 16x^2 + 220x$ do this by looking at the intersection of the line y =800 and the function



/2
If
5.76
(-0.3)

The company must sell 576 radar to make 800 euro in profit

iii. What is the maximum profit the company can make?



/2

The maximum profit is 933.33 euro

Found value for X = 18 as maximum range – 720 euro (0.5)

iv. How many radar signs were made to get this maximum profit? /2

X= 10.000 (x is in hundreds of units sold so 10 x100)

This is when the number of radar sold is 1 000

(j) A factory produces radars signs.

Each radar sign can have two defects that are called fault a and defect b.

A radar sign is chosen at random.

We note A the event "the radar has the defect a" and B the event "the radar has the defect b".

It is assumed that these two events are **independent** and that their probability is P(A)= 0.02 and P(B)=0.01.

The company consider a radar sign is defective when it has at least one of two defects.

i. Calculate the probability that the radar sign is not defective.

A+B =

A only =

B only

N fault

0.98 No fault A — 0.01 fault B; 0.99 no fault B

0.02 fault A — 0.01 fault B; 0.99 No fault B

No defect = 0.98 x 0.99 = 0.9702

/2

ii. Knowing that the sampled radar sign is defective, calculate the probability that it has both defects.

Knowing that it is defective:
P(defect) = 1- no defect   1-0.9702 = 0.0298

Probability it has both faults = 0.02x0.01 = 0.0002

$\frac{0.0002}{0.0298}$ = 0.00671

/2