École Européenne
Paris La Défense
à Courbevoie

MATHEMATICS 3 PERIODS

PART B

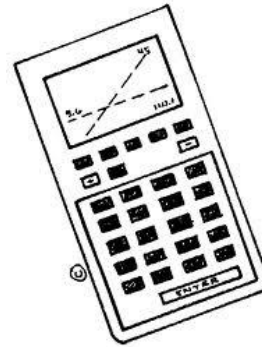**DATE :** 23rd of January 2023, morning

DURATION OF THE EXAMINATION:

2 hours (120 minutes)

AUTHORIZED MATERIAL:

Examination with technological tool:
Numworks calculator

Pencil for the graphs

SPECIFIC INSTRUCTIONS:

This exam paper consists of 6 printed pages, including the cover page (1/6 to 6/6), with an annex page. This annex page may be detached from the paper but must be handed in with the booklet.

Read the questions carefully and show all your workings clearly.

If graphs are used to find a solution, they must be sketched as part of the answer.

Any attempt of cheating will result in the cancellation of your examination on the spot.

Last Name	First name
Mark ……………………. / 50	Final grade

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PART B	
QUESTION B1	**Marks**
Questions 1 and 2 are independent from questions 3 and 4	

The oldest method of preserving food is dehydration. The process consists in using a source of heat to evaporate water from the food. We consider a fresh apricot put in a dryer to be dehydrated.
Before dehydration, this fresh apricot weights 45g of which 85% is water. The process of dehydration is complete when the apricot weights 9g with 25% of water.
At this stage, the fruit is considered as «dried fruits».

1)a) **Calculate** the mass of water contained in the fresh apricot. — 1.5 marks

1)b) **Show** that the dried apricot has 2.25 g of water left. — 1.5 marks

We can model the mass of water depending on the time spent in the dryer by the function $w(t) = 38.25 \times e^{-0.26t}$ with $t \in [0\,;13]$ where $w(t)$ is the weight in grams and t the time in hours.

2)a) **Interpret** the numerical value 38.25. — 1 mark

2)b) **Calculate** the mass of water after 2 hours in the dryer. — 2 marks

2)c) If the apricot spends 8 hours in the dryer, will it be considered as a «dried apricot» ? — 2 marks

2)d) Use technology to **determine** the minimum time for the apricot to be labelled «dried apricot». — 3 marks

An apricot producer buys his trees from three different providers : 35% of the trees come from the tree nursery T_1, 25% from the tree nursery T_2 and the rest from the tree nursery T_3.

Each provider has two qualities of apricot trees : high quality or medium quality. The delivery from the tree nursery T_1 has 80% of high quality trees, the one from T_2 has 50% of high quality trees and the one from T_3 only 30%.

The producer chooses a tree randomly within all deliveries. We consider the following events :

- T_1 : "the tree comes from the tree nursery T_1 "

- T_2 : "the tree comes from the tree nursery T_2 "

- T_3 : "the tree comes from the tree nursery T_3 "

- H : "its a high quality tree "

- M : "its a medium quality tree "

3)a)	**Construct** a tree diagram that models the situation.	2 marks
3)b)	**Calculate** the probability that the tree is a high quality one coming from T_3.	2 marks
3)c)	**Show** that the probability of picking a high quality tree $P(H)$ is 0.525.	2 marks
3)d)	The tree is a high quality one, **calculate** the probability it comes from T_1. Give your answer correct to 3 d.p.	3 marks

A random sample of 10 apricots is selected within the stock of the apricot producer. We assume the stock is large enough so that this selection can be considered as drawing apricots with replacement.

We also assume that the probability for an apricot to be of high quality is the same as for a tree to be a high quality one.

Let X be the random variable counting the number of high quality apricots.

4)a)	**Give** the parameters of the binomial distribution followed by X.	1 mark
4)b)	**Calculate** the probability that exactly 5 apricots are high quality ones. Give your answer correct to 3 d.p.	2 marks
4)c)	**Calculate** the probability that at least 2 apricots are high quality ones. Give your answer correct to 3 d.p.	2 marks

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PART B	
QUESTION B2	**Marks**
Questions 1, 2 and 3 are independent	

For common motor vehicles we consider the two variables *Engine size* (cylinder volume) and *Fuel economy* (number of kilometres travelled for every litre of petrol).

The following data were collected for 10 vehicles.

Car	A	B	C	D	E	F	G	H	I	J
Engine size	1.1	1.2	1.2	1.5	1.5	1.8	2.4	3.3	4.2	5.0
Fuel economy	21	18	19	18	17	16	15	20	14	11

For example, the car A has an *engine size* of 1.1, and a *fuel economy* of 21, meaning it will travel 21 kilometres for 1 litre of petrol.

1)a)	**Construct** a scatter diagram of the above data, the *engine size* being the independent variable. Use the graph paper on the annex page (to be handed in).	2 marks
1)b)	**Describe** the correlation between the two variables.	2 marks
1)c)	Which car gives a fuel economy reading that does not support the general trend ? Note: this outlier is not a recording error, so it cannot be removed.	1 mark
1)d)	Use technology to **calculate** Pearson's correlation coefficient r (correct to 3 d.p.).	2 marks
1)e)	Use technology to **find** the equation of the regression line. Round off the gradient and y-intercept to 1 d.p.	2 marks
1)f)	Use your regression line to **estimate** the fuel economy for an engine size of 2.	2 marks

Cardiac output is an important factor in athletic endurance. The graph shows a stress-test graph of cardiac output (measured in litres/min of blood) versus workload (measured in kg m/min).
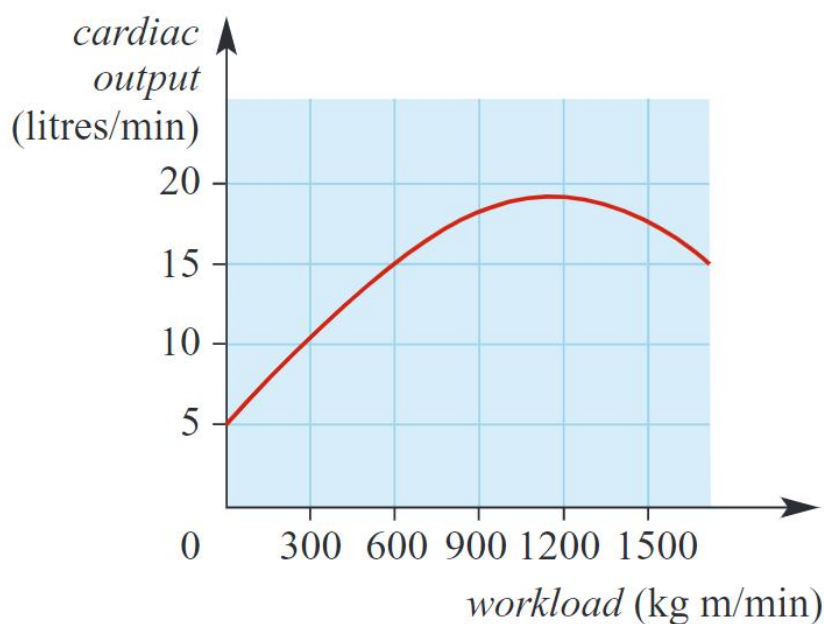
2)a)	**Estimate** the average rate of change of cardiac output with respect to workload as workload increases from 0 to 1500 kg m/min (correct to 4 d.p.).	2 marks
2)b)	**Estimate** the instantaneous rate of change of cardiac output with respect to workload at the point where the workload is 450 kg m/min (correct to 4 d.p.).	3 marks
2)c)	There is a moment when the instantaneous rate of change of cardiac output with respect to workload is equal to zero. **Estimate** for what workload that happens. **Justify** properly your answer.	2 marks

For a new TV show, the main characters are two men, one woman and two girls.

At the end of the first round of castings, there remain 5 actors, 4 actresses and 6 girls.

3)a)	**Determine** how many different casts are possible if there are no restriction.	2 marks
3)b)	One actress and one girl are actually mother and daughter. **Determine** the probability that they would both be chosen for the show (correct to 3 d.p.).	3 marks
3)c)	The producer wants his son, who is one of the 5 remaining actors, to be part of the cast. And in the mean time, two of the 6 girls declined the role. **Determine** how many possible casts there are.	2 marks

ANNEX PAGE (to be handed in)

QUESTION B2, 1)a)

Fuel economy

Engine size