ES Mol 2022-2023
Pre-baccalaureate

Student's name	
Code	

Year	S7
Subject + language	3P Mathematics (Part A) - English
Duration	2 hours (120 minutes)
Teacher	L. Wouters
Date of examination	30/01/2023

| Material authorized	None
Remarks	• This part consists of 10 questions of 5 points each, for a total of 50 pts. • It is essential that the answers be accompanied by the explanations necessary for their preparation. • Responses should highlight the reasoning that leads to the results or solutions. • When graphs are used to find a solution, the response should include sketches of them. • Unless otherwise stated in the question, all points cannot be attributed to a correct answer in the absence of the reasoning and explanations that make it possible to arrive at the results or solutions. • Where an answer is incorrect, however, part of the points may be awarded when an appropriate method and/or correct approach has been used.

Part A

Question A1	5 points

A skateboarder launches himself on a ramp in a skate park. We assume the position of the skater on the ramp can be given by a point with coordinates $(x\,;h(x))$ in the following graph:
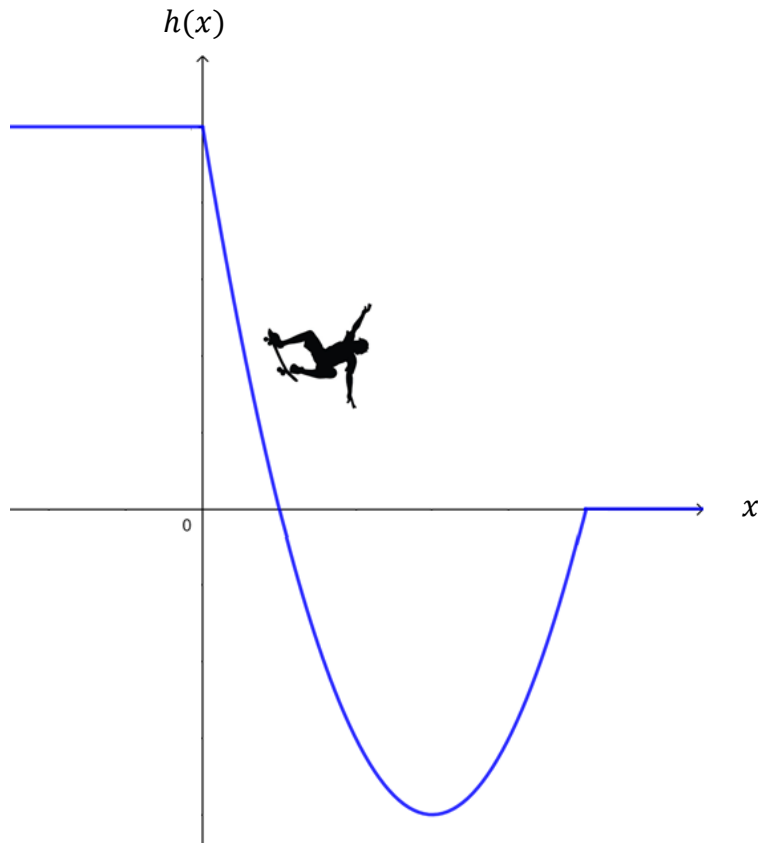
The function h is defined on the interval $[0\,;\,5]$ by:

$$h(x) = x^2 - 6x + 5$$

where $h(x)$ is expressed in metres.

a) **Determine** the height at which the skateboarder launches himself onto the ramp?

b) **Calculate** the value of $h(1)$ and $h(5)$.

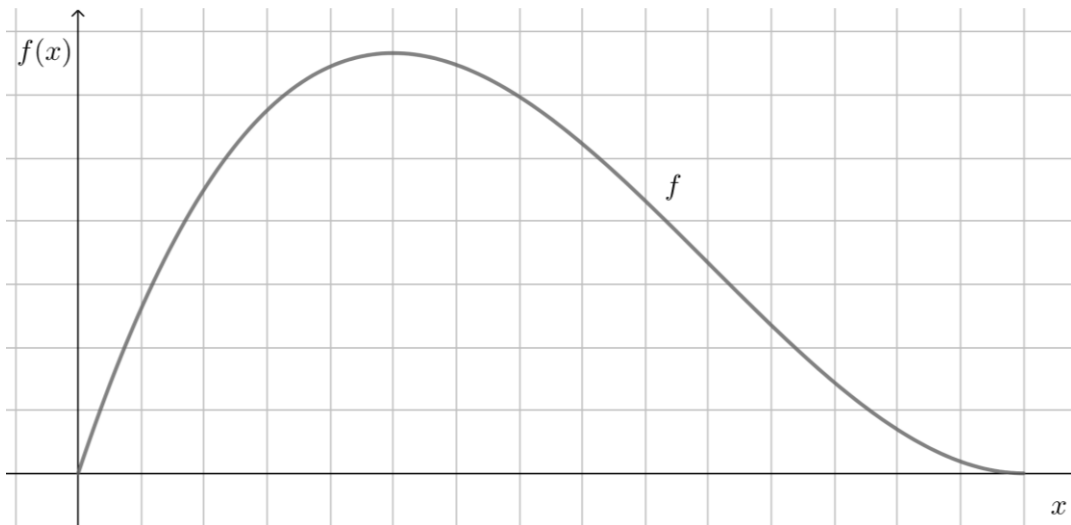c) **Determine** the set of x values for which the skater is below his end point.

The profile of a mountain can be modelled by a function f defined by:

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x \qquad \text{for } 0 < x < 3$$

where x is the distance in meters and $f(x)$ is the height in thousands of meters.

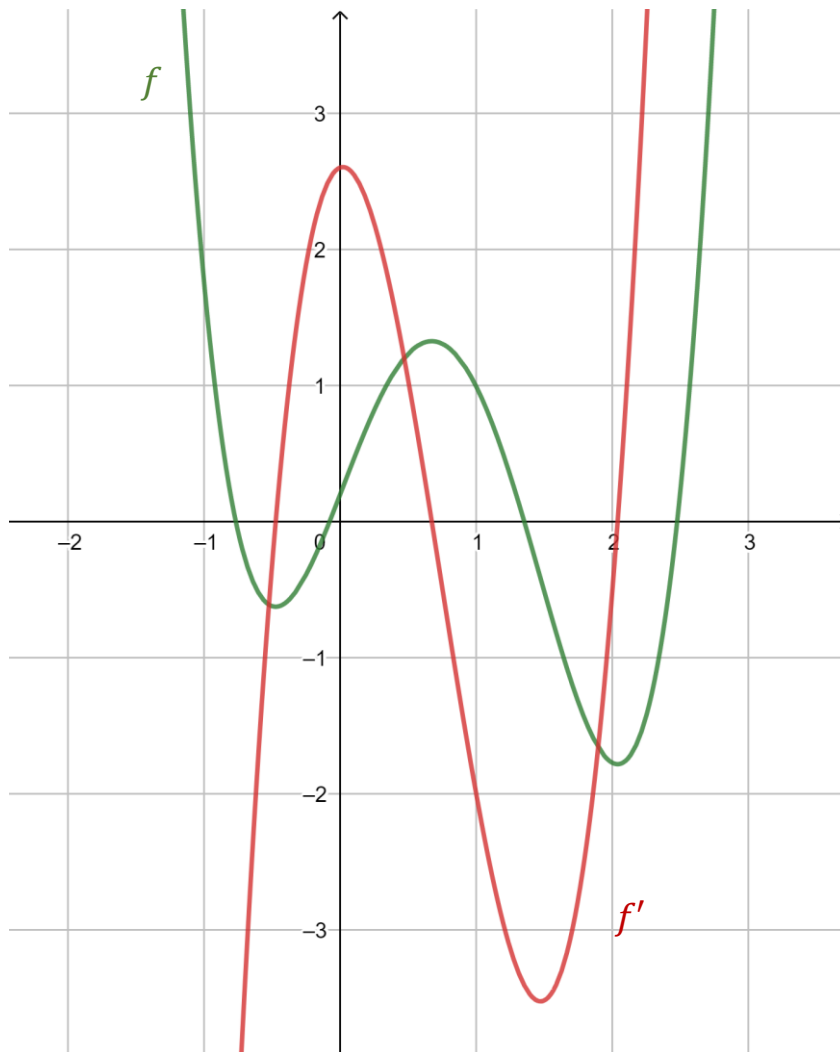We give you the graph that represents this function f :

Determine the height of the mountain, rounded to the nearest hundreds of meters.

The diagram below shows the graph of a function f and that of its derivative function f'.

Determine the equation of the tangent to the graph of $f(x)$ in the point with $x = 1$.
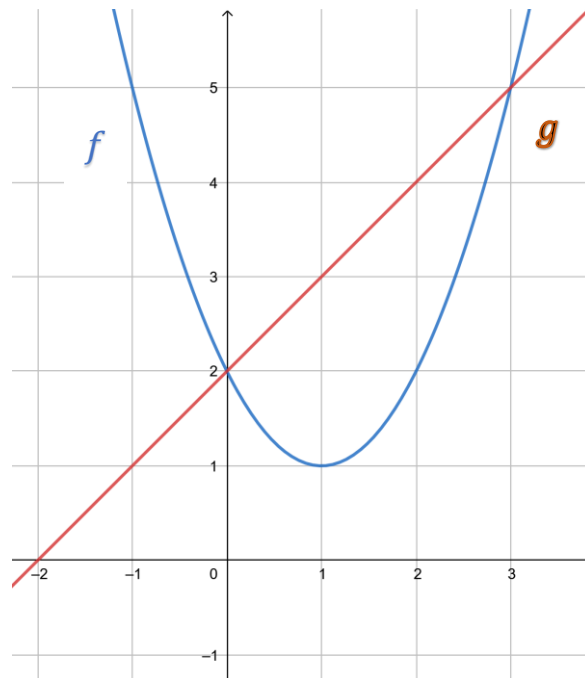
Question A4	5 points

Let f and g be functions that are defined as follows:

$$f(x) = x^2 - 2x + 2 \quad \text{and} \quad g(x) = x + 2$$

and shown in the graph below:

a) **Explain** what $\int_0^3 |f(x) - g(x)| dx$ represents graphically (you can reproduce the graph on your answer sheet and show your answer on the graph).

b) **Calculate** $\int_0^3 |f(x) - g(x)| dx$.

Question A5	5 points

The value of an electric vehicle newly purchased can be modeled by the function:

$$V(t) = 40\,000 \times e^{\ln(0.80)t}$$

where $V(t)$ is the value of the vehicle (in euros), t years after purchase.

a) **Identify** the formula equivalent to the formula $V(t)$ among the following 4 proposals V_1, V_2, V_3 and V_4:

$$V_1(t) = 40\,000 \times \ln(0.80)^t$$

$$V_2(t) = 40\,000 \times 0.80^t$$

$$V_3(t) = 0.80 \times \ln(40\,000)^t$$

$$V_4(t) = 0.80 \times 40\,000^t$$

b) **Determine** the initial purchase price of the vehicle (new).

c) **Calculate** the value of the vehicle one year after purchase.

Question A6	5 points

Since 2004, a company's profits have made a worrisome evolution.

The profits (in hundreds of thousands of euros) of the last 18 years are shown in the graph below:

a) **Give** the names of the two types of fundamental mathematical models that could be used to model this evolution.

b) **Predict** the future year in which profits will again be at a minimum, if the evolution continues in this way.

c) **Interpret** what will happen to this company between now and 2030, if the evolution continues in this way.

Question A7	**5 points**

A waiter, working in a pizzeria, notices that, on average, 40% of the customers are families, the rest are couples.

He also notices that:
- Out of 100 families, 70 leave a tip;
- 4 out of 10 couples leave a tip.

We are interested in the following events:

F: "the table is occupied by a family";
C: "the table is occupied by a couple";
T: "The waiter gets a tip."

a) **Present** all the information of the statement in a probability tree or a two-way table.

b) **Determine** the probability that the table was occupied by a family knowing that the waiter received a tip.

Question A8	5 points
Out of 1500 students at a university, 1200 watch a series during the week, out of which 150 also go to the cinema on weekends. There are 200 students going to the cinema on weekends, without having watched a series during the week. **Determine** if going to the movies on the weekend is dependent on watching a series on weekdays.	

Question A9	5 points

An urn contains 2 red balls and 3 white balls. We draw 3 balls at random.
 	
    a) Please indicate under what condition(s) this situation could be considered as a binomial distribution.
 	
    b) Assuming the condition(s) of a) is/are verified, **calculate** the probability of obtaining only red balls at the end of the 3 draws.	

Question A10	5 points
Let X be a random variable. The table below shows the probability distribution of X:	

x_i	10	20	30	40	50
p_i	a	0.01	0.2	$3a$	0.35

Calculate the expected value of the variable X.

End of Part A