

Pre-Baccalaureate examinations – January 2023



<i>Subject</i>	Mathematics
<i>Date</i>	24/01/2023
<i>Length of examination</i>	2 Hours
<i>Teacher</i>	D Shaw
<i>Pupils</i>	11

Surname:

Name:

PART A : EXAMINATION WITHOUT TECHNOLOGICAL TOOL

Special remarks

- This exam is made up of 10 questions printed on 6 pages, including this one.
- Questions should be answered on a given answer booklet
- Answers must be supported by explanations
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless otherwise indicated, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.

Good luck!

PART A

Marks

1) To sterilise a petri dish before conducting an experiment, it is placed in an oven and the temperature is increased to destroy the bacteria.

The population of bacteria, N , as a function of time, t , in hours is given by the function:

$$N(t) = 1000 \times e^{\ln(0,5)^t}$$

a) This formula could be written in an alternative form. Choose the equivalent formula from the following propositions (no justification required).

$N_1(t) = 1000 \times \ln(0,5)^t$	$N_2(t) = 0,5 \times 1000^t$
$N_3(t) = 1000 \times (0,5)^t$	$N_4(t) = 0,5 \times \ln(1000)^t$

- b) What is the initial population of bacteria before starting the sterilisation?
c) What is the quantity of bacteria after 2 hours?

5 marks

2) A landlord puts up one of his properties for rent. He offers his future tenants two possibilities:

- A. An initial rent of 1000 € with a fixed annual increase of 25 €.
B. An initial rent of 1000 € with an annual increase of 2%.

- a) Calculate the monthly rate of rent to be payed in the second year and in the third year if model A is chosen.
b) Calculate the monthly rate of rent to be payed in the second year and in the third year if model B is chosen.
c) Write a function, $f(x)$, to model the rate at which model A increases over time, where x is the number of years after the signature of the contract.
d) Write a function, $g(x)$, to model the rate at which model B increases over time, where x is the number of years after the signature of the contract.
e) Discuss the most interesting offer over a long term, justifying your choice.

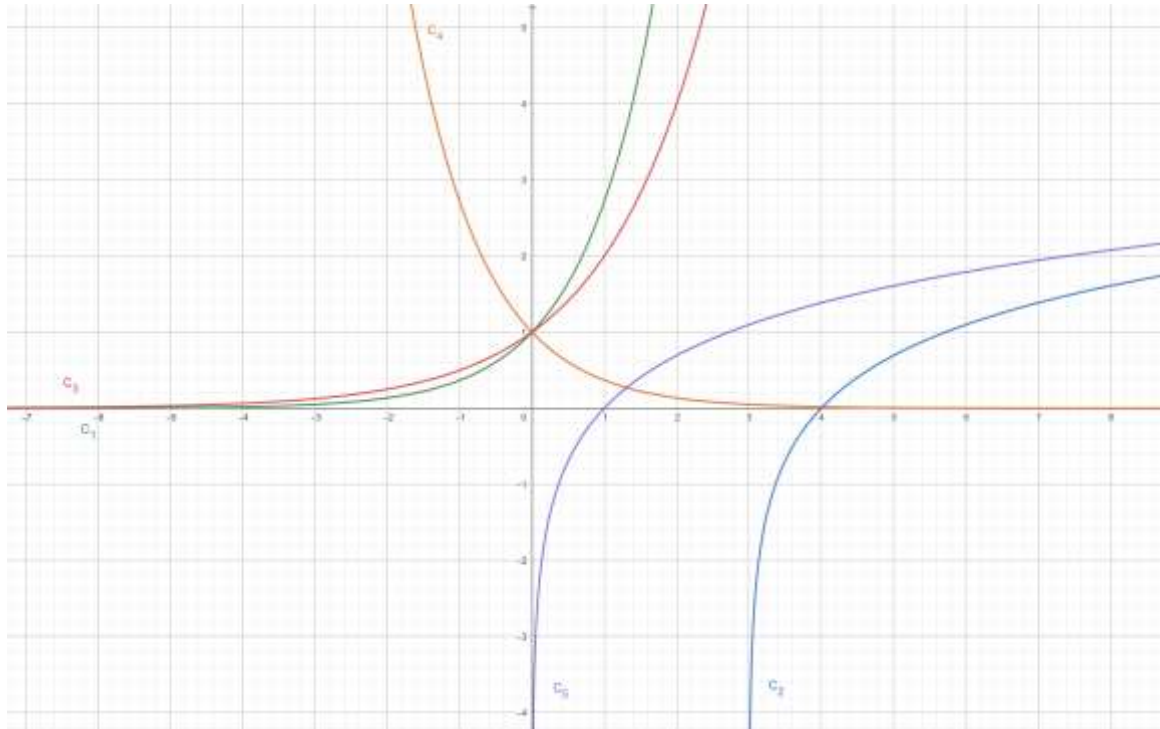
5 marks

PART A

3) Given the functions f, g, h, i and j defined by:

$$f(x) = 2^x \quad g(x) = e^x \quad h(x) = \ln(x) \quad i(x) = \ln(x - 3) \quad j(x) = e^{-x}$$

And their graphic representations:

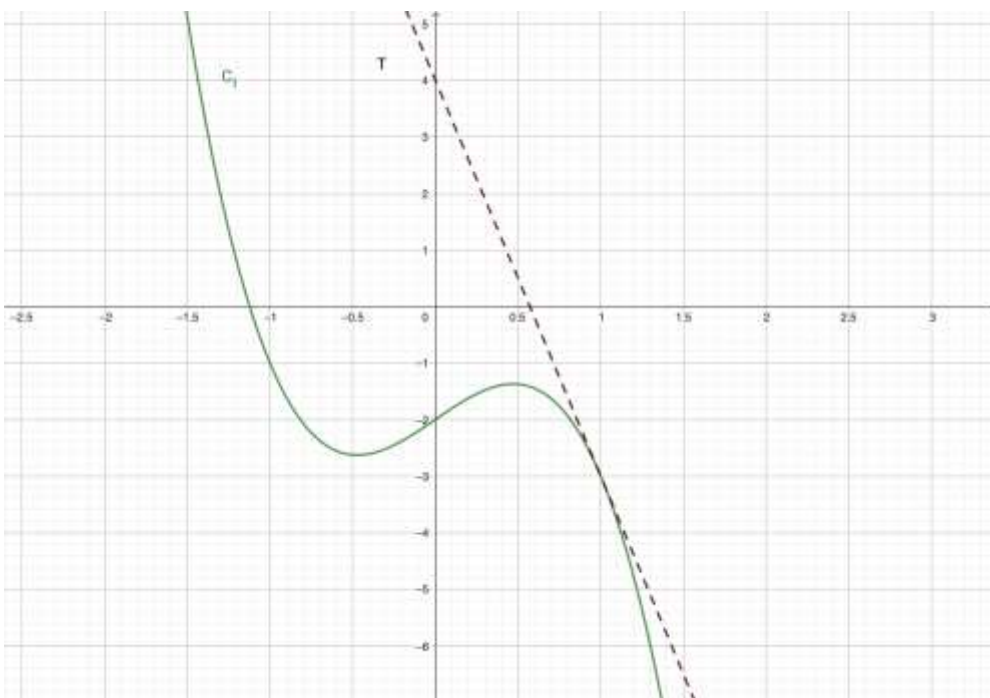


5 marks

Match the function to its corresponding curve. No proof or justification required.

4) Given below is the graph of a function f and the tangent to this function at the point $x = 1$.

Determine the equation of the tangent, T .

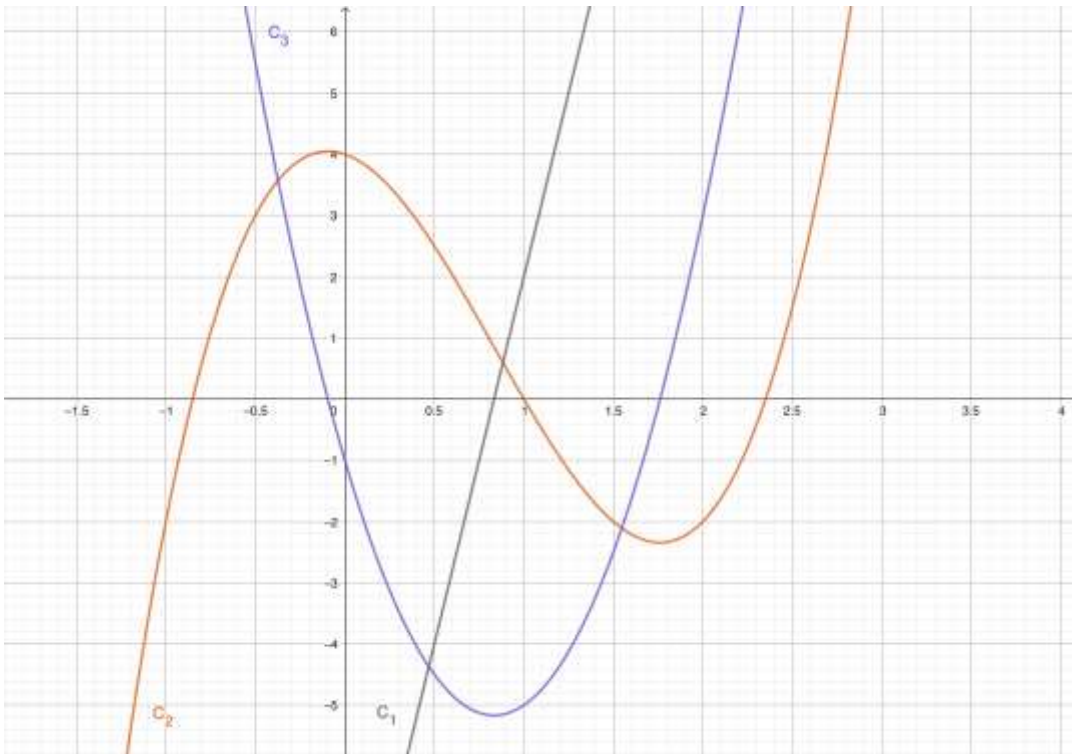


5 marks

PART A

5) On the graph below you are given the curves of a function f , the derivative of this function f' , and one of its primitives F .

5 marks



Identify which curve corresponds to which function and justify your response.

6) The velocity v in $m \cdot s^{-1}$ of an object after t seconds, between $t = 0$ and $t = 6$, is given by the function:

5 marks

$$v(t) = 4t \quad (\text{in metres per second})$$

The acceleration of the object is given by the derivative of the velocity, $v'(t)$

The displacement of the object is given by a primitive, $V(t)$, of the velocity.

- What is the initial speed of the object? Which speed is the object travelling at after 3 seconds?
- Give the expression of the acceleration as a function of time.
- Knowing that the initial position of the object was 10 m, ($V(0) = 10$) give the exact expression of the displacement as a function of time.
- What distance has the object travelled after 6 seconds?

PART A

7)

5 marks



The above curve represents the flow rate, $f(t)$, in litres per minute for filling a container with a capacity of 25L.

- a) Write in terms of $f(t)$ the integral you would use to get the area between the curve and the x-axis for $0 \leq x \leq 5$.
- b) Using the rectangles method with a width of 1, give a left and a right hand estimate of the volume of water poured into the container in the first 5 minutes. Draw on the above diagram.
- c) Interpret the meaning of finding the area between the graph and the horizontal axis on the interval $0 \leq t \leq 5$, in the given context.
- d) Given that the capacity of the container is 25L, will it be full after 5 minutes.

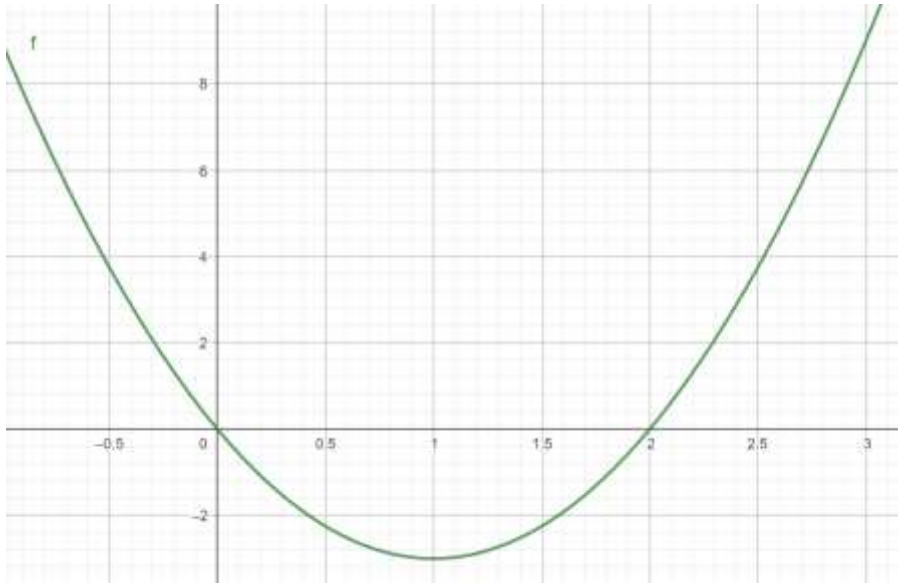
PART A

8) Below is the graph of the function f defined by $f(x) = 3x^2 - 6x$.

5 marks

a) Calculate and anti-derivative of f .

b) Calculate the area bounded by the function and the x-axis for $0 \leq x \leq 3$.



9)

5 marks

a) Calculate the integral:

$$\int_0^1 4e^{5x} dx$$

b) Calculate the integral $F(x)$ of $f(x) = -3x^2 + x + 7$ for which $F(0) = 5$

10) Given the following integrals:

5 marks

$$I = \int_{-2}^2 f(x) dx = 12$$

$$J = \int_2^5 f(x) dx = 3$$

$$K = \int_5^{-2} g(x) dx = 14$$

a) Draw a sketch of the possible graphs of f and g showing the areas represented by the integrals.

b) Calculate the following integrals using the information from integrals I, J and K .

$$A = \int_{-2}^5 f(x) dx$$

$$B = \int_{-2}^5 (f(x) - g(x)) dx$$

$$C = \int_{-2}^2 5f(x) dx$$