## Pre-Baccalaureate examinations - January 2023



| Subject | Mathematics |
| ---: | :---: |
| Date | $24 / 01 / 2023$ |


| Length of examination | 2 Hours |
| ---: | :---: |
| Teacher | D Shaw |
| Pupils | 11 |

## Surname:

Name:

## PART B : EXAM WITH TECHNOLOGICAL TOOL

## Special remarks

- This exam is made up of 2 questions printed on 5 pages, including this one.
- Questions should be answered on a given answer booklet
- Use a different page for each question.
- Answers must be supported by explanations
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless otherwise indicated, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is shown that an appropriate method and/or a correct approach has been used.
- Some of the questions can be answered only with the help of the calculator. The wording of these questions makes this clear. All other questions can be solved with or without the use of the calculator.

| Part B |  |
| :---: | :---: |
| Question B1 ANALYSIS | 25 Marks |
| Use the calculator for questions b), c), d), e), f), h), i), j) and l) <br> Frog and Toad, Arnold Lobel, 1970-1979 <br> 1. The value of a bicycle, in euros, depending on the time $t$ in years, can be described by the function $f$ given $f(t)=750+2250 \cdot e^{-0,2 t}$. |  |
| a) Calculate the value of the bike when new. | 1 mark |
| b) Calculate the value of the bike after one year, and after three years. | 2 marks |
| c) How much does the bicycle lose in value in the first year? Round to the nearest $\epsilon 1$. | 1 mark |
| d) By what percentage has the value of the bicycle decreased after three years? Round to the nearest $1 \%$. | 3 marks |
| e) Solve the equation $f(t)=1500$ and interpret the result. | 3 marks |
| f) Determine value of the bicycle in the long-term based on this model. | 2 marks |
| g) Calculate the derivative $f^{\prime}(t)$. | 2 marks |
| h) Calculate $f^{\prime}(5)$ and interpret the result. | 2 marks |

A $48 \mathrm{~cm}^{3}$ petrol engine is mounted on the bicycle.
The fuel consumption, measured in liters for 100 km , can be calculated as a function of the speed using the function $h(x)=0,04 x+\frac{25}{x}$, where $x$ is the speed in $k m / h$.
i) Graph the function h for $5 \leq x \leq 50$ using the following table of values.

| $x$ | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $h(x)$ |  |  |  |  |  |  |  |  |  |  |

Round the function values to one decimal place.
Graph paper is available.
j) Calculate the petrol consumption at $25 \mathrm{~km} / \mathrm{h}$ in liters for 100 km .
k) Read from your graph, for which speed, $x$, the fuel consumption is the least.
l) Calculate an antiderivative for the function $h$.
m) The bike is pushed and drives off at a speed of $5 \mathrm{~km} / \mathrm{h}$. It is then steadily accelerated to 50 $\mathrm{km} / \mathrm{h}$.
Calculate the integral $\int_{5}^{50} h(x) d x$ with the calculator. Round to the nearest whole number

Hint (this is not a question!): The value $\frac{\int_{5}^{50} h(x) d x}{45}$ is the average fuel consumption per 100 km when accelerating from $5 \mathrm{~km} / \mathrm{h}$ to $50 \mathrm{~km} / \mathrm{h}$.
\(\left.\begin{array}{|l|l|}\hline \& Question B2 ANALYSIS <br>

Marks\end{array}\right]\)| Jane is starting an online business, using a large social media base to promote her website. The |
| :--- |
| weekly visits to her website over the first year can be modelled by the following function: |
| $f(t)=15 \ln (3 t+1), 0 \leq t \leq 52$, where $f(t)$ represents the number of hundreds of visitors |
| her website got and $t$ represents the time measured in weeks with $0 \leq t \leq 52$. |

In reality there were 7820 visitors in the last week of the first year.
i) Which of the two models turns out to be better for predicting this number?

2 marks

One of the objects Jane sells on her website is a mic-stand base. Its profile can be modelled using the function $g(x)=\frac{4}{0.5 x-1.4}$ on the interval $-5 \leq x \leq 2$. Each unit on $x$ and $y$ axis represents 1 cm .
j) Write the integral required to calculate the volume of revolution using the formula

$$
V=\int_{a}^{b} \pi(f(x))^{2} d x
$$

k) Calculate the volume of metal used to make the mic-stand base, in cubic centimeters, to the nearest $\mathrm{cm}^{3}$.

