

MATHEMATICS 3 PERIODS PART A

DATE : 12th June 2023, Afternoon

DURATION OF THE EXAMINATION:

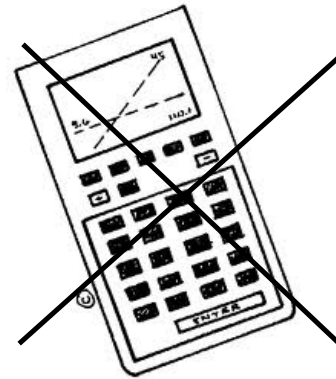
2 hours (120 minutes)

AUTHORISED MATERIAL:

Examination without technological tool

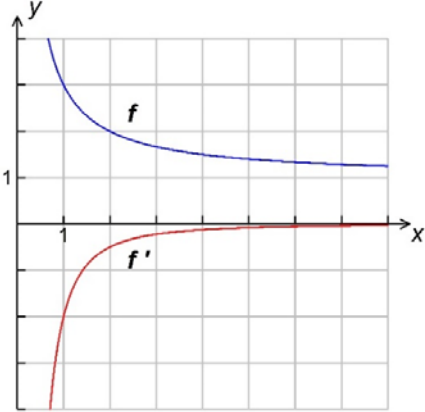
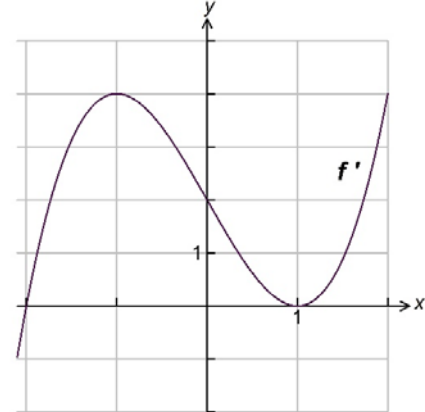
Pencil for the graphs

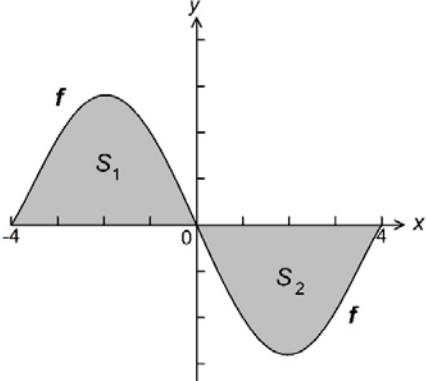
Formelsammlung / Formula booklet / Recueil de formules



SPECIFIC INSTRUCTIONS:

- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is evident that an appropriate method and/or a correct approach has been used.

PART A	Page 1/4	Marks
<p>1) The diagram below shows the graph of a function f and its derivative f'.</p>  <p>Determine and interpret graphically:</p> <p>a) the average rate of change of the function f from $x_1 = 1$ to $x_2 = 2$.</p> <p>b) the rate of change of the function f at $x_1 = 1$.</p> <p>2) Consider a differentiable function f. The figure below shows the graph of its derivative f' for $-2.1 \leq x \leq 2$.</p>  <p>For each of the following statements justify whether it is true or false.</p> <p>a) The function f is decreasing for $-1 \leq x \leq 1$.</p> <p>b) The function f has a minimum at $x = -2$.</p> <p>c) There is a horizontal tangent to the graph of f at the point where $x = 1$.</p> <p>d) The slope of the tangent to the graph of f at the point where it intersects the y-axis is equal to 2.</p> <p>e) The graph of f has three horizontal tangents for $-2.1 \leq x \leq 2$.</p>		<p>2 marks</p> <p>3 marks</p> <p>5 marks</p>

PART A	Page 2/4	Marks
<p>3) Consider the functions f and F defined by</p> $f(x) = 4x^3 + 3x^2 \text{ and } F(x) = x^4 + x^3 + 5 .$ <p>a) Show that F is a primitive function of f.</p> <p>b) Calculate $\int_1^2 f(x)dx$.</p> <p>4) The figure below shows the graph of a function f and two regions S_1 and S_2 bounded by the graph of f and the x-axis. The graph is symmetric with respect to the origin of the coordinate system.</p>  <p>You are given that $\int_{-4}^0 f(x)dx = 7$.</p> <p>a) Interpret the integral $\int_{-4}^0 f(x)dx$ graphically.</p> <p>b) Determine</p> <ol style="list-style-type: none"> $\int_0^4 f(x)dx$. $\int_{-4}^4 f(x)dx$. the area of the region S_2. 		<p>2 marks</p> <p>3 marks</p> <p>2 marks</p> <p>3 marks</p>

EUROPEAN BACCALAUREATE 2023: MATHEMATICS 3 PERIODS

PART A	Page 3/4	Marks
<p>5) A swimming pool is being emptied and the volume of water that remains can be modelled by the function V given by</p> $V(t) = 5000 \cdot 0.60^t, \quad t \geq 0,$ <p>where time t is measured in hours and $V(t)$, measured in litres, is the volume of water, remaining at a time t.</p> <p>Emptying the pool starts at the time $t = 0$.</p> <p>a) Determine the volume of water in the pool at the start and after 1 hour.</p> <p>b) Calculate the percentage rate at which the volume of water decreases per hour.</p> <p>c) Explain what the model tells us about the volume of water remaining after a very long time.</p>		2 marks 2 marks 1 mark
<p>6) a) Calculate in how many ways the letters of the word PARIS can be ordered.</p> <p>b) Calculate how many “words” (not necessarily having a meaning) of 3 different letters you can write using letters of the word PARIS.</p>		2 marks 3 marks
<p>7) A survey of 100 students enrolling at a university, shows that</p> <ul style="list-style-type: none"> • 45 speak English • 40 speak French • 35 speak German • 20 speak both English and French • 23 speak both English and German • 19 speak both French and German • 12 speak all three languages. <p>Using a Venn diagram or otherwise, determine the probability that a randomly selected student from these 100 students speaks only one of these three languages.</p>		5 marks

EUROPEAN BACCALAUREATE 2023: MATHEMATICS 3 PERIODS

PART A	Page 4/4	Marks
<p>8) Applicants for jobs in a large company must sit an aptitude test. They are either</p> <ul style="list-style-type: none"> • accepted with a probability of $\frac{1}{5}$ or • rejected with a probability of $\frac{1}{2}$ or • retested with a probability of $\frac{3}{10}$. <p>When they are retested, there are just two outcomes, accepted with a probability of $\frac{2}{5}$ or rejected with a probability of $\frac{3}{5}$.</p>		
a) Draw a tree diagram to illustrate the outcomes.		2 marks
b) Determine the probability that a randomly selected applicant is accepted.		3 marks
<p>9) A biased coin is thrown several times. At each throw, the probability of getting a head is $\frac{1}{3}$.</p>		
a) Is this a Bernoulli process? Justify your answer.		2 marks
b) The coin is thrown 3 times. Calculate the probability of getting exactly 2 heads.		2 marks
c) The coin is thrown 60 times. Calculate the expected value for the number of heads.		1 mark
<p>10) A machine produces steel balls. The diameter of the balls is normally distributed with mean $\mu = 18.0$ mm and standard deviation $\sigma = 0.5$ mm. A ball is selected at random.</p>		
a) Determine the probability that its diameter is between 17.0 mm and 19.0 mm.		1 mark
b) Determine the probability that its diameter is between 17.0 mm and 18.5 mm.		2 marks
c) A batch of 400 steel balls is selected at random from this production and the diameter of each ball is measured. If the diameter of a ball is less than 17.0 mm, it will be rejected. Estimate how many balls will be rejected.		2 marks