

MATHEMATICS 3 PERIODS PART A

RESERVE 1

DATE: 19 June 2023, Afternoon

DURATION OF THE EXAMINATION:

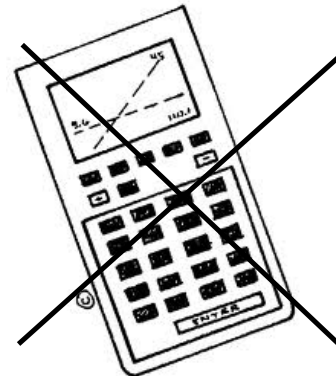
2 hours (120 minutes)

AUTHORISED MATERIAL:

Examination without technological tool

Pencil for the graphs

Formelsammlung / Formula booklet / Recueil de formules

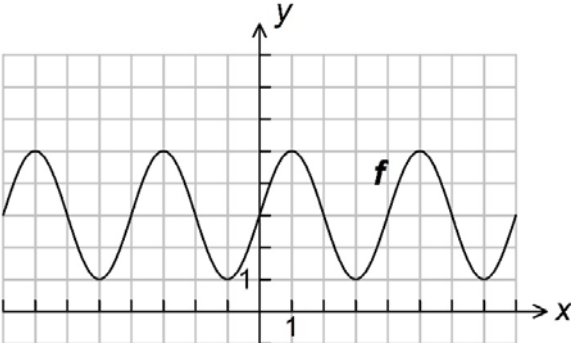


SPECIFIC INSTRUCTIONS:


- Answers must be supported by explanations.
- They must show the reasoning behind the results or solutions provided.
- If graphs are used to find a solution, they must be sketched as part of the answer.
- Unless indicated otherwise, full marks will not be awarded if a correct answer is not accompanied by supporting evidence or explanations of how the results or the solutions have been achieved.
- When the answer provided is not the correct one, some marks can be awarded if it is evident that an appropriate method and/or a correct approach has been used.

PART A	Page 1/5	Marks
<p>1) The diagram below shows the graph of a function f and the tangent at the point P where $x = 2$.</p> <div data-bbox="527 415 1036 919" style="text-align: center;"> </div> <p>a) Determine $f(2)$ and $f'(2)$ graphically. 2 marks</p> <p>b) Determine an equation of the tangent to the graph of f at the point P. 2 marks</p> <p>c) Solve the equation $f'(x) = 0$ graphically. 1 mark</p> <p>2) Consider the function f where $f(x) = \frac{1}{2}x^2 + 1$.</p> <p>In a coordinate system sketch the graph of f, and draw 4 rectangles to approximate the region bounded by the graph of f and the x-axis for $0 \leq x \leq 4$.</p> <p>Use these rectangles to determine an approximate value of the area of this region. 5 marks</p>		

PART A	Page 2/5	Marks																																					
<p>3) Consider a differentiable function f. The figure below shows the graph of its derivative f' for $0 \leq x \leq 7$.</p> <div style="text-align: center; margin: 10px 0;"> </div> <p>Which one of the tables below describes the variation of the function f for $0 \leq x \leq 7$? Explain your answer.</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p>A.</p> <table border="1" style="border-collapse: collapse; width: 150px;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">3.5</td> <td style="padding: 5px;">7</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td colspan="3" style="text-align: center;"> </td> </tr> </table> </div> <div style="text-align: center;"> <p>B.</p> <table border="1" style="border-collapse: collapse; width: 150px;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">7</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td colspan="4" style="text-align: center;"> </td> </tr> </table> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p>C.</p> <table border="1" style="border-collapse: collapse; width: 150px;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">7</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td colspan="4" style="text-align: center;"> </td> </tr> </table> </div> <div style="text-align: center;"> <p>D.</p> <table border="1" style="border-collapse: collapse; width: 150px;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">7</td> </tr> <tr> <td style="padding: 5px;">$f(x)$</td> <td colspan="3" style="text-align: center;"> </td> </tr> </table> </div> </div>			x	0	3.5	7	$f(x)$				x	0	2	5	7	$f(x)$					x	0	2	5	7	$f(x)$					x	0	2	7	$f(x)$				5 marks
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<p>4) On a farm the wheat production P in kg per hectare can be modelled by</p> $P(t) = 6000 \cdot e^{-\ln(2) \cdot t},$ <p>where t is the number of years after 2022.</p> <p>a) Calculate the wheat production in 2023 according to this model.</p> <p>b) Determine in what year the wheat production will be 1500 kg per hectare according to this model.</p> <p>5) The figure below shows the graph of the function f defined by</p> $f(x) = a \cdot \sin(b \cdot x) + d,$ <p>where the parameters a, b and d are integers.</p>  <p>a) Determine the values of a and d.</p> <p>b) Determine the period p of f and calculate the value of b.</p> <p>6) A study at a certain university found that</p> <ul style="list-style-type: none"> • 70 % of the students own a computer • 40 % of the students owning a computer also own a car. • 55 % of the students do not own a car. <p>A student from this university is selected at random. Consider the following two events:</p> <p>Event O: "the student owns a computer" Event A: "the student owns a car".</p> <p>Are the events O and A independent? Justify the answer.</p>		<p>2 marks</p> <p>3 marks</p> <p>2 marks</p> <p>3 marks</p> <p>5 marks</p>

PART A	Page 4/5	Marks																
<p>7) 800 cats were tested with a new test for a feline virus. The cats were also tested with an older version of the test, which is slower and more expensive, but totally accurate.</p> <p>The following results were obtained:</p> <table border="1" data-bbox="391 478 1127 789"> <thead> <tr> <th></th> <th>Having the virus</th> <th>Not having the virus</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>New test positive</td> <td>63</td> <td></td> <td></td> </tr> <tr> <td>New test negative</td> <td></td> <td>717</td> <td></td> </tr> <tr> <td>Total</td> <td>68</td> <td></td> <td>800</td> </tr> </tbody> </table> <p>Complete the table and copy it to your answer sheet.</p> <p>Using the table, calculate the following probabilities:</p> <ul style="list-style-type: none"> • The probability of getting a negative result with the old test and a positive result with the new test. • The probability that the new test gives a correct result. • The probability that a cat is tested negative with the new test, given that it has the virus. 			Having the virus	Not having the virus	Total	New test positive	63			New test negative		717		Total	68		800	5 marks
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<p>8) Leila goes out into her family's garden to pick a few apples. Only one out of three apples is ok to eat. The rest of the apples are worm eaten. Leila randomly picks 4 apples.</p> <p>a) This may be seen as a Bernoulli process. Explain why.</p> <p>b) Calculate the probability that Leila picks exactly 2 apples that are ok to eat.</p> <p>c) Calculate the probability that at least 1 of the 4 apples is ok to eat.</p>		<p>1 mark</p> <p>2 marks</p> <p>2 marks</p>																

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<p>9) The 1984 “California Avocado Society” study of more than two hundred twenty-five million avocados determined that the weight of avocados is normally distributed with a mean of 215 grams and a standard deviation of 5 grams.</p> <p>Only avocados weighing between 210 grams and 225 grams are considered fit for sale.</p> <p>a) Show that 81.5 % of avocados are fit for sale.</p> <p>b) Determine the probability that an avocado weighs more than 215 grams, given that it is fit for sale. Give the answer as a fraction of integers.</p> <p>10) A manufacturer produces titanium bicycle frames. The bicycle frames are tested before use and on average 7 % of them are found to be faulty. A cheaper manufacturing process is introduced, and the manufacturer wishes to check whether the proportion of faulty frames has increased. A random sample of 18 bicycle frames is selected and it is found that 4 of them are faulty. The manufacturer will carry out a hypothesis test at a 5 % significance level to see if the proportion of faulty bicycle frames has increased.</p> <p>a) State a suitable null hypothesis H_0 and an alternative hypothesis H_a for the test.</p> <p>The random variable X describes the number of faulty bicycle frames in a sample of 18 bicycles. The table below shows the value of $P(X \geq k)$ for $k = 1, 2, 3, 4, 5$ and 6 for a probability of 0.07 of having a faulty frame.</p> <table border="1" data-bbox="297 1591 1219 1703"> <thead> <tr> <th>k</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>$P(X \geq k)$</td> <td>0.729</td> <td>0.362</td> <td>0.127</td> <td>0.0333</td> <td>0.00665</td> <td>0.00105</td> </tr> </tbody> </table> <p>b) Will the null hypothesis be rejected? Can we assume that the percentage of faulty bicycle frames has increased? Explain your answer.</p>	k	1	2	3	4	5	6	$P(X \geq k)$	0.729	0.362	0.127	0.0333	0.00665	0.00105	 <p>3 marks</p> <p>2 marks</p> <p>2 marks</p> <p>3 marks</p>
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