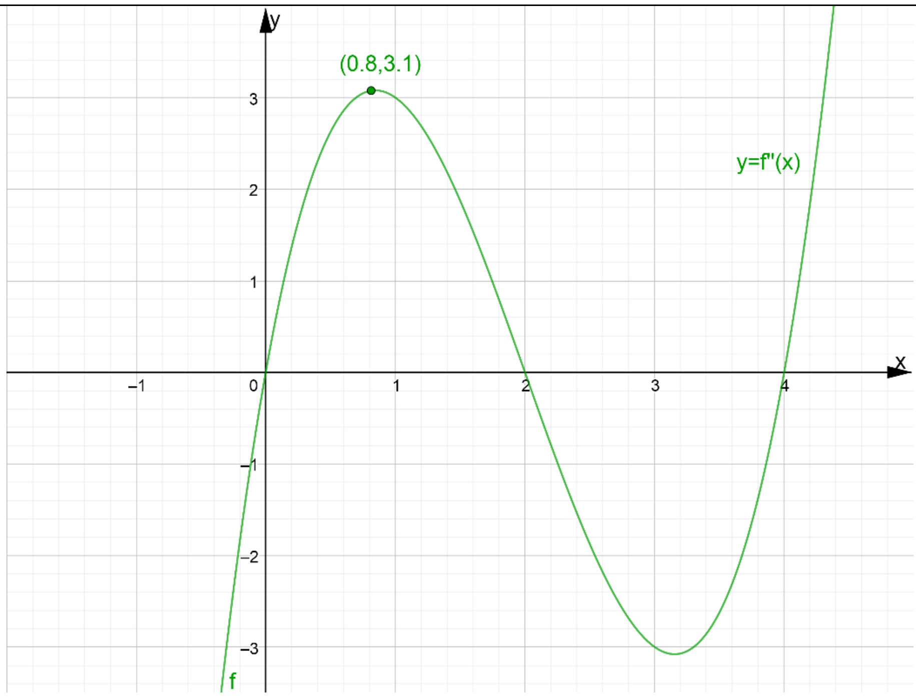
Part A Answers

	Part A	Points			
		KC	M	PS	I
S1	Given the function f, where $f(x) = \ln(3x-2)$, **determine** the equation of the tangent to the graph of f when $x = 1$.				
	$f(1) = 0$ $f'(1) = 3$ $0 = 3(1) + c$ $c = -3$ $y = 3x - 3$	3	1		
S2	**Determine** the complex solutions to the equation: $z^2 = 3i$. **Give** your answers on the form $z = re^{i\theta}$ where $\theta \in]-\pi, +\pi]$.				
	$z = (3i)^{\frac{1}{2}} = (3cis\left(\frac{\pi}{2}\right))^{\frac{1}{2}} = \sqrt{3}cis\left(\frac{\pi}{4}\right) = \sqrt{3}e^{\frac{i\pi}{4}}$	3	2		
S3	Given the function $f(x) = \frac{2x-1}{x-1}$. Let f^{-1} be the inverse function of f. **Solve** the equation $f^{-1}(x) = 2$.				
	Determine the inverse function: $y = \frac{2x-1}{x-1}$ $y \cdot x - y = 2x + 1$ $y \cdot x - 2x + 1 = y$ $x \cdot (y-2) = y - 1$ $x = \frac{y-1}{y-2}$ exchange x and y: $y = f^{-1}(x) = \frac{x-1}{x-2}$ $f^{-1}(x) = \frac{x-1}{x-2} = 2$ $x - 1 = 2x - 4 \quad \Rightarrow \quad x = 3$		2	1	
S4	A strictly increasing arithmetic sequence (a_n) and a geometric sequence (b_n) have the same first term, where $a_1 = b_1 = 2$. Additionally, both (a_n) and (b_n) have the same third term. That is $a_3 = b_3$ The sum of the first three terms of the arithmetic sequence is 4 greater than the sum of the first three terms of the geometric sequence. **Determine** the formula for the nth term of both (a_n) and (b_n).				
	$a_1 = b_1 = 2$ $a_3 = b_3 \iff a_1 + 2d = b_1 \cdot q^2 \iff 2 + 2d = 2 \cdot q^2 \quad \|:2$ $1 + d = q^2 \quad \|-1$ $d = q^2 - 1 \quad (I)$ $s_{a,3} = s_{b,3} + 4 \iff 3a_1 + 3d = b_1\left(1 + q + q^2\right) + 4$ $\iff 6 + 3d = 2\left(1 + q + q^2\right) + 4 \quad \|-4$ $2 + 3d = 2 + 2q + 2q^2 \quad (II)$	2	2	3	

	(I) in (II): $2+3(q^2-1)=2+2q+q^2$ $2+3q^2-3=2+2q+q^2 \quad \vert -2-2\mathrm{q}-2\mathrm{q}^2$ $q^2-2q-3=0$ pq-formula: $q=1\pm\sqrt{1+3} \quad \Rightarrow \quad q_1=3, \qquad q_2=-1$ $\Rightarrow \quad d_1=3^2-1=8, \qquad d_2=(-1)^2-1=0$ d_2 can be excluded, as the sequence (a_n) is strictly increasing according to the precondition. $\Rightarrow \begin{matrix}(a_n)=2+(n-1)\cdot 8=8n-6 \\ (b_n)=2\cdot 3^{n-1}\end{matrix}$				
S5	A continuous random variable *X* has a density function given by a formula: $$f(x)=\begin{cases}0 & ,x<0 \\ a\cdot e^{-ax} & ,x\geq 0\end{cases}$$ We know that $P(X<1)=\frac{1}{2}$. **Show** that $a=\ln 2$.				
	$$\int_0^1 ae^{-ax}dx={}_0^1[-e^{-ax}]=-e^{-a}+1$$ $$-e^{-a}+1=\frac{1}{2}$$ $$e^{-a}=\frac{1}{2}$$ $$e^a=2$$ $$a=\ln(2)$$		2	3	
S6	Given is the graph of the second derivative f'' of a function (see figure below). **Decide** which of the following statements are true and which are false. **Justify** your answer. a) The graph of f is concave for $-0{,}5<x<2$. b) The graph of f has an inflection point at $x=0$. c) If $f'(0)=0$, then the graph of f has an inflection point with a horizontal tangent at $x=0$.				

					
	a) False. For -0.5<x<2 f''(x) assumes values greater than and less than zero. b) True. f'' is equal to zero at x=2 with change of sign from + to -. c) True. If f'(0)=0, f''(0)=0, and second derivative changes its sign at x=0,then we have a stationary point of inflection.		 1 2	2 1	
E1	A drone manufacturer tests new types of drones at a local athletics field. Drone A moves along the path given by the equation: $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}, t \geq 0$$ The time t is in seconds and distance is measured in meters. a) **Find** the position of drone A after 6 seconds. b) **Determine** how long it will take the drone A to reach the point $(25,33,60)$. c) **Calculate** the speed of the drone A. Give your answer in a simplest surd form. d) There is an observer watching drone A from the point $(13,53,0)$. **Calculate** the shortest distance between the drone A and the observer, and the time when it occurs. Drone B takes off from the point $(9,11,0)$ and moves at 7 m/s in the direction $\begin{pmatrix} 1 \\ 1.5 \\ 3 \end{pmatrix}$. e) **Show** that the equation describing the position of the drone B is: $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}, t \geq 0$$ f) **Find** the point at which the paths of the drones A and B intersect. g) **Decide** whether the drones will collide at this point. **Justify** your answer.				
	a) t=6, so (28,37,72) b) $\begin{pmatrix} 25 \cdot \\ 33 \\ 60 \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}$, t=5 c) $\sqrt{3^2+4^2+12^2} = \sqrt{169} = 13\ m/s$	1 1	1 1 1	 1	

	d) $\begin{pmatrix}-3+3t\\-40+4t\\12t\end{pmatrix}\circ\begin{pmatrix}3\\4\\12\end{pmatrix}=0, so\ t=1$ and $\left\|\begin{pmatrix}0\\-36\\12\end{pmatrix}\right\|=\sqrt{1440}=12\sqrt{10}$ e) $\left\|\begin{pmatrix}1\\1.5\\3\end{pmatrix}\right\|=\sqrt{12.25}=3.5\quad but\ the\ speed\ of\ the\ drone\ B\ is\ two\ times\ bigger,$ $so\ the\ velocity\ vector\ of\ the\ drone\ B\ must\ be\ \begin{pmatrix}2\\3\\6\end{pmatrix}$ f) $\begin{pmatrix}10\\13\\0\end{pmatrix}+t_A\begin{pmatrix}3\\4\\12\end{pmatrix}=\begin{pmatrix}9\\11\\0\end{pmatrix}+t_B\begin{pmatrix}2\\3\\6\end{pmatrix}$, point of intersection *(13,17,12)* g) $No\ because\ t_A=1s\ and\ t_B=2s$		2 2	1 1 2	 1
E2	Two players, A and B alternately and independently flip a fair coin. The first player to get a head wins. Assume player A flips first. a) **Write down** the probability that A wins in a first throw. b) **Calculate** the probability that A wins in a third throw. c) **Determine** the probability that A obtains the first head.				
	a) $\frac{1}{2}$ b) $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$ c) Player one can win at the first, the third, the fifth throw., so P(A)=$\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\ldots\ldots\ldots$ P(A)=$\frac{1}{2}+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^5_{\ldots\ldots}$ It leads the infinite geometric sequence with $a_1=\frac{1}{2}$ and $q=\frac{1}{4}$, so $s_\infty=\frac{a}{1-q}=\frac{\frac{1}{2}}{\frac{3}{4}}=\frac{2}{3}$			3	2