MATHEMATICS 5 PERIODS
PART B

DATE: DD/MM/YYYY

DURATION OF THE EXAMINATION: 120 minutes

EXAMINATION WITH TECHNOLOGICAL TOOL

AUTHORISED MATERIAL:

Technological tool

Formula Booklet

Notes:

- As this is a sample paper the cover page is likely to change.
- This sample paper should only be used to see how questions can be created from the syllabus focusing on competences rather than strictly on content.
- The keywords found in the syllabus are highlighted in bold to help the candidate see which competency the question is focusing on and thus helping in answering the question.

PART B

Question 1/4 — Marks

Tom and Simon play a board game. Each time Tom manages to move his piece around the board he gets 5 points. Each time Simon manages to move his piece around the board he gets 10% of the previous amount. They both start with 10 points.

a) **Calculate** Tom's total score after moving around the board 20 times. — 2

b) **Write** in terms of n the formula $\boldsymbol{T(n)}$ for Tom's score after $\boldsymbol{n}$ moves around the board. — 2

c) If you know that Simon's score after n moves around the board could be modelled with a geometric sequence, **explain** the use of the formula: — 2

$$\boldsymbol{S(n) = 11 \cdot 1.1^{n-1}}$$

d) Simon and Tom have been around the board the same number of times. Simon's score has just moved ahead of Tom's. — 3

Find how many times have they been around the board.

Tom challenges Simon to a dice game. Two fair six-sided die are rolled and the sum of scores is noted. For a sum less than 6 Simon receives 10 cents, for a sum between 6 and 9 Simon loses 5 cents, and for the sum bigger or equal 10 Simon receives 30 cents. The winnings are governed by the probability distribution shown below, where the random variable N is the sum of scores.

N	$\boldsymbol{n < 6}$	$\boldsymbol{6 \leq n \leq 9}$	$\boldsymbol{n \geq 10}$
Winnings n	10 cents	−5 cents	30 cents
$\boldsymbol{P(N = n)}$	a	$\boldsymbol{\frac{20}{36}}$	b

e) **Show**, that $\boldsymbol{a} = \frac{\mathbf{10}}{\mathbf{36}}$ et $\boldsymbol{b} = \frac{\mathbf{6}}{\mathbf{36}}$. — 2

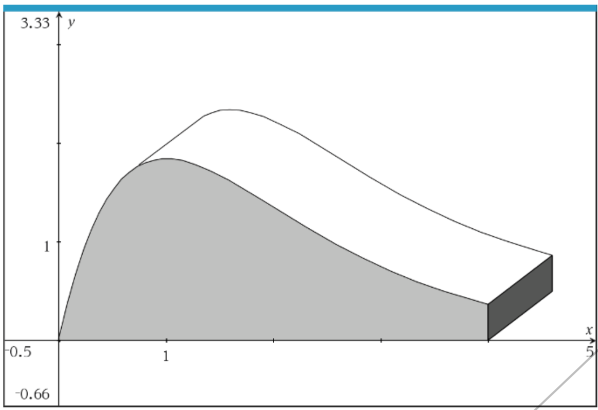
f) **Calculate** the expected value of Simon's winnings in this game and comment if it is worth Simon playing. — 2

g) A game is said to be fair if the expected value is 0. — 2

Determine how many cents should be lost for the sum between 6 and 9 to make this game fair.

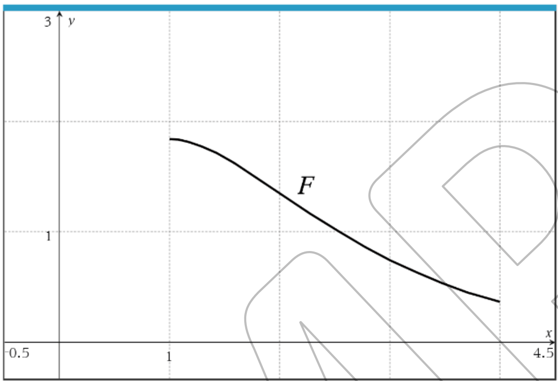
PART B	
Question 2/4	**Marks**

A kids' play area manufacturer wants to offer its customers a new model of slide. They create a diagram of the proposed slide in an oblique projection:



The profile of this slide is measured in meters and can be modeled by the function

$F(x) = (ax - b)e^{-x}$, for $1 \le x \le 4$, where a and b are two parameters. The function F was drawn below.



a) It is planned that the tangent to the function F at the point where $x = 1$ would be horizontal. — 3

Determine the value of the parameter b.

b) It is also planned that the top of the slide will be at 1.85 meters. — 2

Determine the value of the parameter a.

The profile of the wall is finally modeled by $F(x) = 5x \cdot e^{-x}$.

c) **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m². — 2

d) **Determine** the point on the slide where the gradient is greatest. — 3

PART B

Question 3/4	Marks

Optical smoke detectors contain a photocell as an important component. A factory produces photocells for this purpose. A controller automatically checks photocells and rejects those that are faulty. On average he is 86% accurate. However, the accuracy of the controller is found to vary - sometimes he detects a higher percentage of faulty photocells and sometimes a lower percentage. The controller's accuracy is found to be modelled by a normal distribution with a standard deviation of 5%.

a) **Find** the probability that the controller is less than 85% accurate. — 1

b) $\frac{9}{10}$ of the time the controller is less than x% accurate. **Determine** x. — 2

c) Given that, on a particular day, the controller is less than 90% accurate, **find** the probability that he is more than 85% accurate. — 2

Two types of optical smoke detector are being tested for reliability. The higher the probability of an alarm being triggered the more reliable it is.

Type A contains a single photocell and is triggered when this photocell is activated.

Type B contains three photocells and is triggered if at least two of the three photocells are activated.

The probability of a photocell being activated in the presence of smoke is p. The probability of both types of alarm being triggered is calculated for different values of p.

$P(A_p)$ is the probability of type A being triggered when the probability is p,

$P(B_p)$ is the probability of type B being triggered when the probability is p.

d) **Complete** the table below. — 4

p	0.3	0.5	0.7
$P(A_p)$	0.3	0.5	0.7
$P(B_p)$			
More reliable type			

e) **Determine** for what value of p does type B become more reliable than type A. — 2

f) **Show** that, in terms of p, $P(A_p) = p$ and $P(B_p) = -2p^3 + 3p^2$. — 4

g) **Explain** the meaning of the following function R in relation to the context of the question. **Explain** what is calculated in lines (1) to (3) and **interpret** the result. — 3

$$R: p \longmapsto R(p) = -2p^3 + 3p^2 - p$$

$$(1)\quad R'(p) = -6p^2 + 6p - 1$$

$$(2)\quad R'(p) = 0 \implies p_1 \approx 0{,}79$$

$$(3)\quad R''(p_1) < 0$$

PART B	
Question 4/4	**Marks**
Given are the plane $E: 2x_1 - x_2 + 3x_3 = 5$ and for each $a \in \mathbb{R}$ a straight line: $$g_a: \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ a \\ 2 \end{pmatrix}$$	
a) **Determine** the coordinates of the intersection of the straight line g_a with the plane E in terms of a.	4
b) **Find** for which value of a is there no solution. **Interpret** the result geometrically.	3