Part B Answers

	Part B	Points			
		KC	M	PS	I

1

Tom and Simon play a board game. Each time Tom manages to move his piece around the board he gets 5 points. Each time Simon manages to move his piece around the board he gets 10% of the previous amount. They both start with 10 points.

a) **Calculate** Tom's total score after moving around the board 20 times.

b) **Write** in terms of n the formula $\boldsymbol{T(n)}$ for Tom's score after $\boldsymbol{n}$ moves around the board.

c) If you know that Simon's score after n moves around the board could be modelled with a geometric sequence, **explain** the use of the formula:

$$\boldsymbol{S(n) = 11 \cdot 1.1^{n-1}}$$

d) Simon and Tom have been around the board the same number of times. Simon's score has just moved ahead of Tom's.
Find how many times have they been around the board.

Tom challenges Simon to a dice game. Two fair six-sided die are rolled and the sum of scores is noted. For a sum less than 6 Simon receives 10 cents, for a sum between 6 and 9 Simon loses 5 cents, and for the sum bigger or equal 10 Simon receives 30 cents. The winnings are governed by the probability distribution shown below, where the random variable N is the sum of scores.

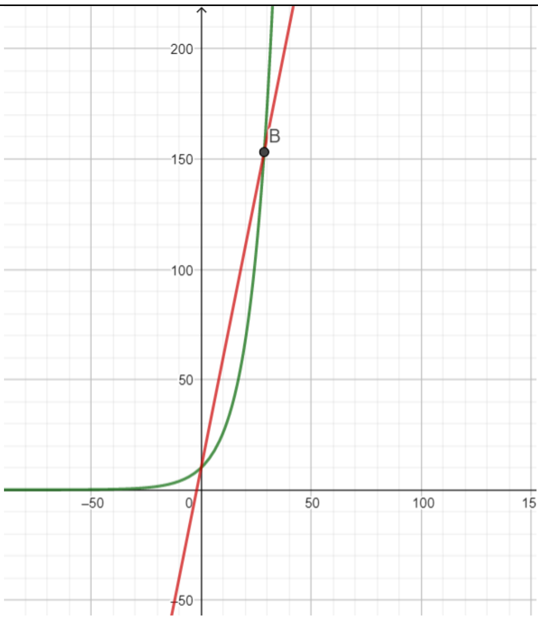
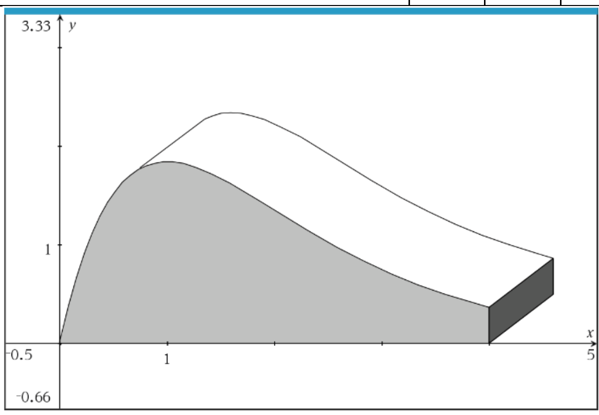
N	$\boldsymbol{n < 6}$	$\boldsymbol{6 \leq n \leq 9}$	$\boldsymbol{n \geq 10}$
Winnings n	10 cents	–5 cents	30 cents
$\boldsymbol{P(N = n)}$	a	$\boldsymbol{\frac{20}{36}}$	b

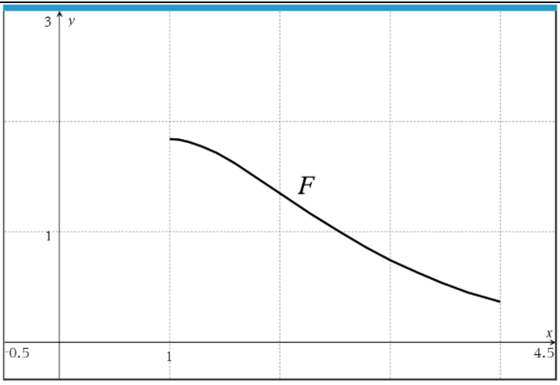
e) **Show**, that $\boldsymbol{a = \frac{10}{36}}$ et $\boldsymbol{b = \frac{6}{36}}$.

f) **Calculate** the expected value of Simon's winnings in this game and comment if it is worth Simon playing.

g) A game is said to be fair if the expected value is 0.
Determine how many cents should be lost for the sum between 6 and 9 to make this game fair.

Answer	KC	M	PS	I
a) Calculate Tom's score after moving around the board 20 times. $c = 15 + 19 \cdot 5$ $\rightarrow 110$	2			
b) Write in terms of n the formula T(n) for Tom's score after n moves around the board. T(n)=5n+10	1	1		
c)		2		
d) Sketch the graph of T(n) and S(n) in the same coordinate system.		2	1	

	 What is the least number of moves around the board for the Simon's score to exceed the Tom's score. B = Intersect(S, T, (28.63, 153.16)) → (28.63, 153.16) Answer: 29 moves e) $\boldsymbol{a = P(X=2)+P(X=3)+P(X=4)+P(X=5)}$ $= 1/36 + 2/36 + 3/36 + 4/36 = 10/36$ $\boldsymbol{b = 1 - 10/36 - 20/36 = 6/36}$ f) E(X)= $\frac{\mathbf{10}}{\mathbf{36}} \times 10 + \frac{\mathbf{20}}{\mathbf{36}} \times (-5) + \frac{\mathbf{6}}{\mathbf{36}} \times 30 = 5$ Yes, it is worth playing, because the expected value is a positive number. g) E(X)= $\frac{\mathbf{10}}{\mathbf{36}} \times 10 + \frac{\mathbf{20}}{\mathbf{36}} \times x + \frac{\mathbf{6}}{\mathbf{36}} \times 30 = 0,$ x=-14	1 1	1 1 1	1	
	A kids' play area manufacturer wants to offer its customers a new model of slide. They create a diagram of the proposed slide in an oblique projection:  The profile of this slide is measured in meters and can be modeled by the function $F(x) = (ax - b)e^{-x}$, for $1 \leq x \leq 4$, where a and b are two parameters. The function F was drawn below.				



a) It is planned that the tangent to the function F at the point where $x = 1$ would be horizontal.
Determine the value of the parameter b.

b) It is also planned that the top of the slide will be at 1.85 meters.
Determine the value of the parameter a.

The profile of the wall is finally modeled by $F(x) = 5x \cdot e^{-x}$.

c) **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m^2.

d) **Determine** the point on the slide where the gradient is greatest.

Answer				
a/b) 1 Derivative$((a\,x - b)\,e^{-x}, x, 1)$ $\rightarrow\ a\,e^{-x} + b\,e^{-x} - a\,x\,e^{-x}$				
2 Solve$(a\,e^{-1} + b\,e^{-1} - a\,e^{-1} = 0, b)$ $\rightarrow\ \{b = 0\}$	2	1		
3 Solve$(a\,e^{-1} = 1.85, a)$ $\rightarrow\ \left\{a = \frac{37}{20}\,e\right\}$	1	1		
c) **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m^2. $\begin{cases} u = 5x, & v' = e^{-x} \\ u' = 5, & v = -e^{-x} \end{cases}$ $\int 5xe^{-x}\,dx = -5xe^{-x} + 5\int e^{-x}\,dx$ $-5xe^{-x} - 5e^{-x} + c$ $\int_0^4 5xe^{-x}\,dx = [-5xe^{-x} - 5e^{-x}]_0^4 =$ $(-5 \cdot 4e^{-4} - 5e^{-4}) - (-510e^0 - 5e^0) =$ $-\frac{20}{e^4} - \frac{5}{e^4} + 5=$ $(5 - \frac{25}{e^4})$ m^2	2			
d) x=2 A = Min(f1′(x), 0, 4) $\rightarrow$ (2, -0.68)		1	2	

3 Optical smoke detectors contain a photocell as an important component. A factory produces

photocells for this purpose. A controller automatically checks photocells and rejects those that are faulty. On average he is 86% accurate. However, the accuracy of the controller is found to vary - sometimes he detects a higher percentage of faulty photocells and sometimes a lower percentage. The controller’s accuracy is found to be modelled by a normal distribution with a standard deviation of 5%.

a) **Find** the probability that the controller is less than 85% accurate.

b) $\frac{9}{10}$ of the time the controller is less than x% accurate. **Determine** x.

c) Given that, on a particular day, the controller is less than 90% accurate, **find** the probability that he is more than 85% accurate.

Two types of optical smoke detector are being tested for reliability. The higher the probability of an alarm being triggered the more reliable it is.
Type A contains a single photocell and is triggered when this photocell is activated.
Type B contains three photocells and is triggered if at least two of the three photocells are activated.
The probability of a photocell being activated in the presence of smoke is p. The probability of both types of alarm being triggered is calculated for different values of p.
$P(A_p)$ is the probability of type A being triggered when the probability is p,
$P(B_p)$ is the probability of type B being triggered when the probability is p.

d) **Complete** the table below.

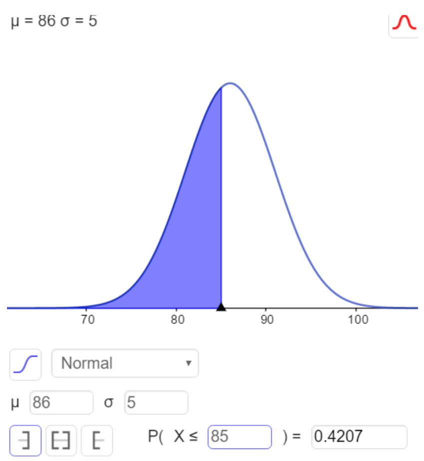
p	0.3	0.5	0.7
$P(A_p)$	0.3	0.5	0.7
$P(B_p)$			
More reliable type			

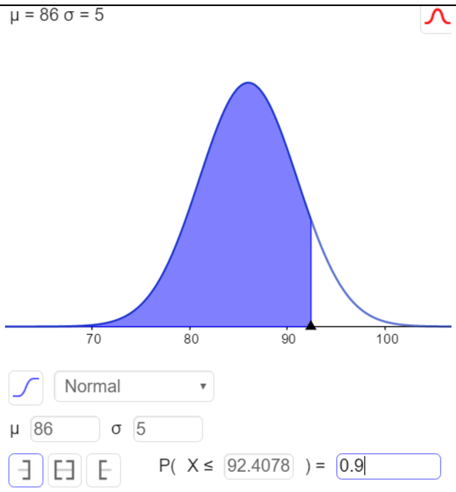
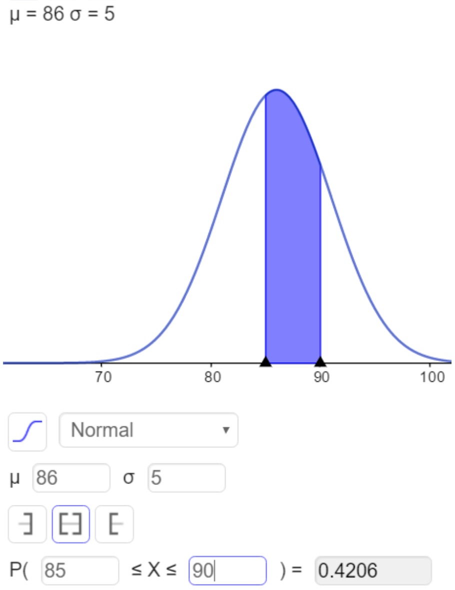
e) **Determine** for what value of p does type B become more reliable than type A.

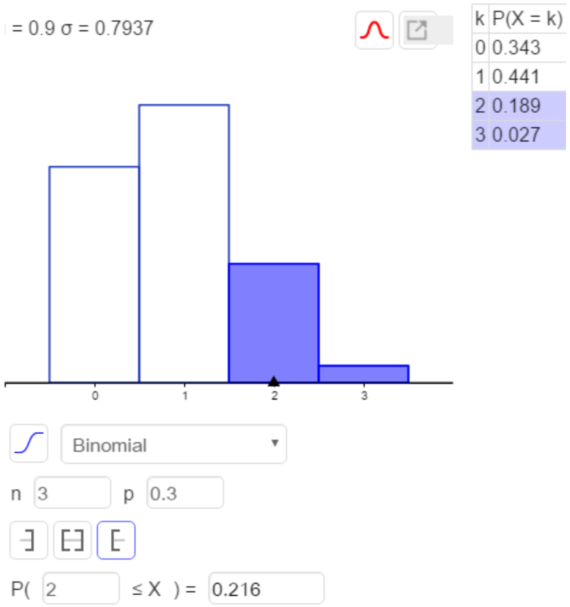
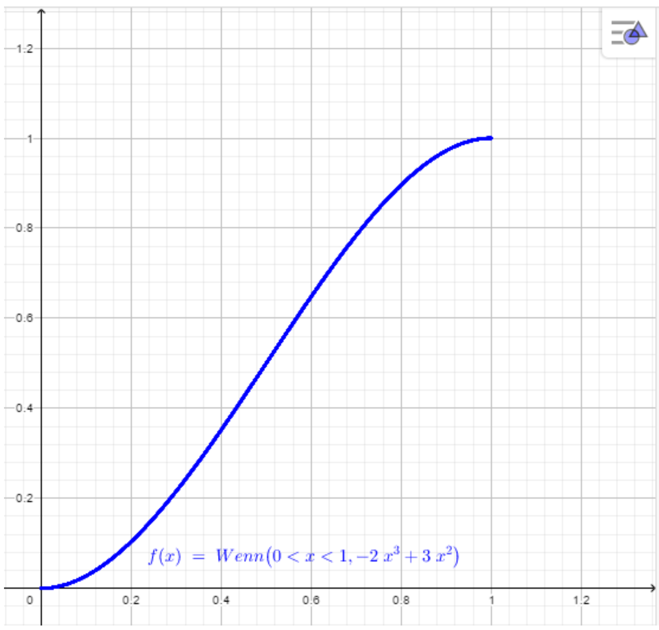
f) **Show** that, in terms of p, $P(A_p) = p$ and $P(B_p) = -2p^3 + 3p^2$.

g) **Explain** the meaning of the following function R in relation to the context of the question. **Explain** what is calculated in lines (1) to (3) and **interpret** the result.

$$R: p \longmapsto R(p) = -2p^3 + 3p^2 - p$$
$$(1)\quad R'(p) = -6p^2 + 6p - 1$$
$$(2)\quad R'(p) = 0 \implies p_1 \approx 0{,}79$$
$$(3)\quad R''(p_1) < 0$$

a) 	1 1	1		

	b) 	1	1		
	c) 				
	d) The random variable y: “Number of photocells showing a reaction” is assumed to be binomially distributed with n = 3 and p. $P(Y \geq 2) = P(Y = 2) + P(Y = 3)$ $= \binom{3}{2} \cdot p^2 \cdot (1-p)^1 + \binom{3}{3} \cdot p^3$ $\Longrightarrow P(0{,}3) = 0{,}216;\ \ P(0{,}5) = 0{,}500,$ $P(0{,}7) = 0{,}784$ <u>GeoGebra:</u> (shown for P(0,3) only)		1	3	

			1	2 3	
	e) The results show that the reliability of smoke detectors is improved if $p > 0.5$, but if $p < 0.5$, the probability of triggering an alarm is reduced by this measure, thus worsening the reliability. If $p = 0.5$, the reliability of an alarm remains consistently good. f) $P(p) = P(Y \geq 2) = \binom{3}{2} \cdot p^2 \cdot (1-p)^1 + \binom{3}{3} \cdot p^3$ $= \frac{3!}{2!\,(3-2)!}(p^2 - p^3) + \frac{3!}{3!\,(3-3)!} \cdot p^3$ $= 3 \cdot (p^2 - p^3) + 1 \cdot p^3$ $= 3p^2 - 3p^3 + p^3 = -2p^3 + 3p^2$ 				
	g) The function R is the difference function of P and p. R indicates for which p the reliability P(p) with three photocells is greater than the reliability p of a single photocell. Equation (1): The derivative of R is determined. Equation (2): The zero of the derivative of the function R is			1	2

	calculated. It is located at the position $p_1 \approx 0{,}79$. Equation (3): The sign of the second derivative function at position p_1 is checked. R'' has a negative sign there. In this way, an extremum is calculated for the difference function R, namely a maximum, since $R''(p)$ is negative. If $p \approx 0.79$ is chosen, the difference function $R(p)$ has the highest value, i.e. the reliability of the message is greatest here.				
4	Given are the plane $E: 2x_1 - x_2 + 3x_3 = 5$ and for each $a \in \mathbb{R}$ a straight line: $g_a: \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ a \\ 2 \end{pmatrix}$ a) **Determine** the coordinates of the intersection of the straight line g_a with the plane E in terms of a. b) **Find** for which value of a is there no solution. **Interpret** the result geometrically.				
	a) g_a in E: $2t - (1 + ta) + 3(1 + 2t) = 5$ $2t - 1 - ta + 3 + 6t = 5$ $(8 - a) \cdot t = 3$ $t = \frac{3}{8-a}$ for $a \neq 8$ $\Rightarrow S_a\left(\frac{3}{8-a} \middle\vert 1 + \frac{3a}{8-a} \middle\vert 1 + \frac{6}{8-a}\right)$		2	2	
	b) There is no solution for $a = 8$. In this case the direction vector of the straight line g_8 is perpendicular to the normal vector of the plane. $\begin{pmatrix} 1 \\ 8 \\ 2 \end{pmatrix} \circ \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 0$ $\Rightarrow$ The line and the plane are parallel. (a) g_a in E: Löse(2 (0 + λ) − (1 + λ A) + 3 (1 + 2 λ) = 5, λ) → $\left\{\lambda = \frac{-3}{A - 8}\right\}$ Intersection point Sa: Vektor((0 + λ, 1 + λ A, 1 + λ · 2)) Ersetze: $\begin{pmatrix} \frac{3}{-A+8} \\ 3 \cdot \frac{A}{-A+8} + 1 \\ 1 + \frac{6}{-A+8} \end{pmatrix}$ No solution for A=8, as the denominator becomes 0. Skalarprodukt({1, 8, 2}, {2, −1, 3}) → 0 Geometric interpretation: line and plane are parallel			2	1