MATHEMATICS 5 PERIODS

EXAMPLES OF FINAL ASSESSMENT

FRENCH / ENGLISH

The structure and content of the final assessment is given in the syllabus:

> **[All] topics from S6 […] could potentially be included.**
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> The final Baccalaureate written exam will consist of two parts, one part with and one part without a technological tool. Each part will make up 50% of the total 100 marks and will be 120 minutes long. All topics from the S6 and S7 5 Period courses can be examined in the final written Baccalaureate exam.
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> The overall weighting of the competences will be set by the **Mathematics BAC matrix**. The weightings are designed to ensure meaningful marks across the range of performances. Performances should be distinguishable not only quantitatively ("more or less of the same") but also qualitatively ("different levels of attainment"). This will make the paper accessible to all pupils while providing more challenging problems to obtain higher marks. […]
>
> The part without the technological tool will consist of **six short response questions and two longer questions that require more extended mathematical thinking.** These extended questions could be structured, giving pupils greater guidance, or more open, requiring pupils to develop a suitable strategy for solving the problem.
>
> The part with the technological tool will consist of a smaller number of longer, structured questions that allow pupils to explore a given context in more depth. In general, the level of thinking will increase as a pupil works through the questions. The technological tool will need to be used to fully answer this paper, though this does not exclude the possibility that some questions could be fully answered without the use of the tool.

The structure of the papers should therefore not be as rigid as it was with the previous syllabus. Questions can cover any subject, in any order, and one question can cover more than one part of the syllabus, as is the case for some questions in these examples.

Example paper 2 – part B – solutions

Question B1

Gabriella is playing with her remote-controlled toy car. The following equation describes the path of the car:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 1 \end{pmatrix} + t \begin{pmatrix} -12 \\ 5 \end{pmatrix}$$

The distance units are metres, and the time is in minutes.

1. Write down the initial position of the car.
2. Calculate the position of the car after 15 seconds.
3. Compute the speed of the car.

Grandma is watching Gabriela from point $P(-1,-6)$

4. Find the shortest distance from point P to the path of the car.

The edge of the cliff is at the point $\left(0,\dfrac{23}{3}\right)$ and Grandma walks in that direction with velocity vector $\begin{pmatrix} 3 \\ 41 \end{pmatrix}$.

5. After how many minutes will the car reach the edge of the cliff?
6. Will Grandma be able to catch the car before it falls down the cliff if she starts moving at the same time as the car? Explain your answer.

Solution:

1. $\begin{pmatrix} 16 \\ 1 \end{pmatrix}$
2. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 1 \end{pmatrix} + \dfrac{1}{4} \begin{pmatrix} -12 \\ 5 \end{pmatrix} = \begin{pmatrix} 13 \\ 2.25 \end{pmatrix}$
3. $\sqrt{(-12)^2+5^2}=13\left[\dfrac{m}{s}\right]$
4. Line perpendicular to the path of the car: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} + s \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

   $\begin{cases} -1+5s=16-12t \\ -6+12s=1+5t \end{cases}$

   $t=1, P'(4,6)$

   $\left|\overrightarrow{PP'}\right|=\sqrt{5^2+12^2}$

   The shortest distance is 13 metres.
5. $0=16+t(-12)$ yields $t=\dfrac{4}{3}[min]$.
6. $0=-1+3g$ yields $g=\dfrac{1}{3}[min]$. So Grandma should be able to catch the car.

Question B2

1. A contractor must carry out work for a public body. If they do not complete the work on time, they will have to pay a daily penalty: 100 € on the first day, 110 € on the second day, and so on with a daily increase of 10 € a day.
   Let u_n be the penalty on the n-th day. Thus, the first term in sequence u is $u_1=100$.
   a. State the nature and characteristics of sequence u.
   b. Explain why $u_n=90+10n$ for all values of integer n.
   c. On what day would the daily penalty amount to 220 €?
   d. What total amount of penalty would the contractor have paid after 20 days of delay?
2. On another construction site, the penalty for delay is 80 euros on the first day and then increases by 10% each day. Let v_n be the amount of the penalty on day n in this case.
   a. Compute the values of the first three terms v_1, v_2 and v_3.
   b. Explain why $v_n=80\cdot 1.10^{n-1}$ for all values of integer n.
   c. What is the total amount of penalty the contractor would have paid after 20 days of delay?
3. From which day onwards does the amount of the daily penalty in the second case exceed that of the first case?

Solution :

1. a. u is an arithmetic sequence with first term $u_1=100$ and common difference 10.

b. $u_n=100+10(n-1)=100+10n-10=90+10n$

d. We solve the equation $100+10(n-1)=220$, which yields $n=13$, so this will happen on the 13th day.

e. The total amount is the sum $\sum_{k=1}^{20} u_k$.

$$\sum_{k=1}^{20} u_k=\frac{100+290}{2}\cdot 20=3900.$$

So the contractor would have paid a total of 3 900 €.

2. a. $v_1=80$, $v_2=80\cdot 1.10=88$ and $v_3=88\cdot 1.10=96.8$.

b. The first term is $v_1=80$. Then, from one term to the next, we add 10%, which comes to multiplying by 1.10. Therefore, v is a geometric sequence with first term $v_1=80$ and common ratio 1.10.

So $v_n=80\cdot 1.10^{n-1}$ for all values of integer n.

d. The total amount is the sum $\sum_{k=1}^{20} v_k$.

$$\sum_{k=1}^{20} v_k=80\cdot\frac{1-1.10^{20}}{1-1.10}\approx 4582.$$

So the contractor would have paid a total of 4 582 €.

3. We have to compare the total amounts. This can be done using a spreadsheet, or the table of values of functions $f(n)=\frac{100+100+10(n-1)}{2}\cdot n$ and $g(n)=80\cdot\frac{1-1.10^{n-1}}{1-1.10}$.

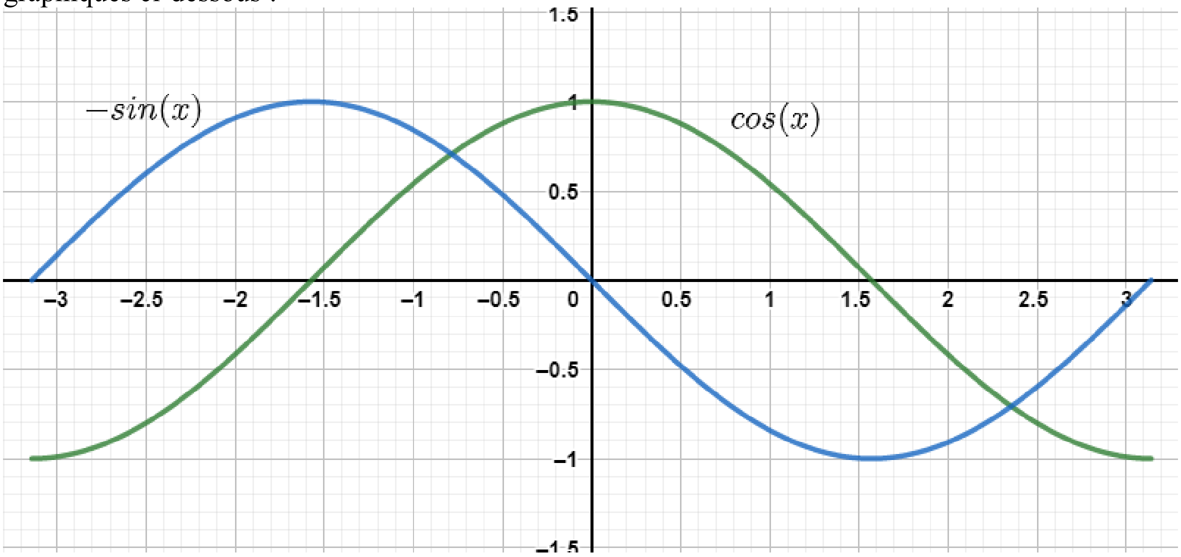
We find that this will happen on the 16th day.

n	u_n	sum	v_n	sum

1	100	100	80	80
18	270	3330	404.3576	3647.934
19	280	3610	444.7934	4092.727
20	290	3900	489.2727	4582

Question B3

1. On considère les fonctions $x \mapsto \cos x$ et $x \mapsto -\sin x$ sur $[-\pi\,;\pi]$ et leurs représentations graphiques ci-dessous :



Justifier que les seules solutions de l'équation $\cos x+\sin x=0$ sur $[-\pi\,;\pi]$ sont $\frac{-\pi}{4}$ et $\frac{3\pi}{4}$.

2. Soit f la fonction définie sur $[-\pi\,;\pi]$ par : $f(x)=e^x\cdot\sin x$

On note C_f sa courbe représentative dans un repère.

a. Déterminer les variations de la fonction f sur $[-\pi\,;\pi]$, en précisant l'abscisse, la valeur et la nature de chaque extremum.

b. Déterminer une équation de la tangente à la courbe C_f au point d'abscisse $\frac{\pi}{2}$.

c. Sur quel intervalle C_f est-elle entièrement située au-dessus de chacune de ses tangentes ? Justifier.

d. En utilisant deux intégrations par parties successives, calculer la valeur exacte de l'intégrale :

$$\int_{-\pi}^{\pi} f(x)\,dx$$

Solution :

1. D'après le graphique, l'équation n'a que deux solutions, qui correspondent aux points d'intersection des deux courbes. Or $x=\frac{-\pi}{4}$ ou $x=\frac{3\pi}{4}$ vérifient l'équation. Ce sont donc bien les deux seules solutions.

2. a. On calcule $f'(x)$:

$$f'(x)=e^x ¿$$

$f'(x)$s'annule et change de signe en $\frac{-\pi}{4}$ (négative puis positive) et $\frac{3\pi}{4}$ (positive puis négative), donc f admet un minimum en $\frac{-\pi}{4}$ (d'ordonnée $f\left(\frac{-\pi}{4}\right)=\frac{-\sqrt{2}}{2}e^{\frac{-\pi}{4}}$) et un maximum en $\frac{3\pi}{4}$ (d'ordonnée $f\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}}$).

b. Equation de la tangente :

$$y=f'\left(\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)=e^{\frac{\pi}{2}}\left(x+1-\frac{\pi}{2}\right)$$

c. C_f est au-dessus de chacune de ses tangentes quand elle est convexe, c'est-à-dire quand $f''(x)>0$.

On calcule $f''(x)$

$$f''(x)=2e^x\cos x$$

Donc $\cos x>0$ donc $\frac{-\pi}{2}<x<\frac{\pi}{2}$

Donc C_f est au-dessus de ses tangentes sur l'intervalle $¿-\frac{\pi}{2};\frac{\pi}{2}¿$.

d.

$$\int_{-\pi}^{\pi} f(x)\,dx=\int_{-\pi}^{\pi} e^x\sin x\,dx$$

On pose $u(x)=e^x$ donc $u'(x)=e^x$, et $v'(x)=\sin x$ donc $v(x)=-\cos x$

Ainsi $\int_{-\pi}^{\pi} e^x\sin x\,dx=\left[-e^x\cos x\right]_{-\pi}^{\pi}+\int_{-\pi}^{\pi} e^x\cos x\,dx$

On calcule ensuite $\int_{-\pi}^{\pi} e^x\cos x\,dx$

On pose $u(x)=e^x$ donc $u'(x)=e^x$, et $v'(x)=\cos x$ donc $v(x)=\sin x$

Donc

$$\int_{-\pi}^{\pi} e^x\sin x\,dx=\left[-e^x\cos x\right]_{-\pi}^{\pi}+\left[e^x\sin x\right]_{-\pi}^{\pi}-\int_{-\pi}^{\pi} e^x\sin x\,dx$$

Ainsi

$$2\int_{-\pi}^{\pi} e^x\sin x\,dx=\left[-e^x\cos x\right]_{-\pi}^{\pi}+\left[e^x\sin x\right]_{-\pi}^{\pi}=\left[-e^x\cos x\right]_{-\pi}^{\pi}$$

Donc

$$\int_{-\pi}^{\pi} e^x\sin x\,dx=\frac{\left[-e^x\cos x\right]_{-\pi}^{\pi}}{2}=\frac{e^{\pi}-e^{-\pi}}{2}$$

Question B4

A company is conducting a study into the relationship between the experience and salary of their staff. The experience and salaries of 12 employees were tabulated.

Experience x (years)	0	2	4	6	8	10	12	14	16	18	20	22
Salary y (€)	4200	4800	4600	5000	5200	5600	5650	5660	5500	6000	5831	6200

1. One of the following correlation coefficients fits these data. Which is it?

$$r_1 \approx 0.95,\ r_2 \approx -0.95 \text{ or } r_3 = 1\,?$$

Explain without referring to any computations.

2. Compute the coordinates of the average point for these data, to the nearest integer.

3. The equation of regression line with the method of the least squares is $y = a + bx$, where

$$b = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \quad \text{and} \quad a = \bar{y} - b\bar{x}.$$

Use the information given below to compute the values of coefficients a and b. Give answers to 2 d.p.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
0	-11	121
2	-9	81
4	-7	49
6	-5	25
8	-3	9
10	-1	1
12	1	1
14	3	9
16	5	25
18	7	49
20	9	81
22	11	121

$$\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) = 45009$$

4. Use the linear model $f(x) = 78.7x + 4488$ to estimate the salary of an employee with 40 years of experience.

The salaries of the employees of this company are normally distributed with mean $\mu = 5353$ and standard deviation $\sigma = 553$.

5. Mr. Smith, an employee of this company is paid 6459 €. What proportion of the employees of this Company are paid less that Mr. Smith?

6. Compute the probability that an employee's salary is greater than 7 636 € and comment your question for question **5.**

In another company, the salaries are normally distributed with standard deviation $s = 620$.

7. Knowing that the probability that an employee's salary is greater than 5000 € is approximately 0.107, find the mean salary in that company. Write your answer to the nearest whole number.

Solution:

1. Values of x and y are generally increasing, so r is positive. The correlation is not perfect, so r can't be equal to 1. Therefore, the only possible value is r_1.

2. $\bar{x} = 11$ and $\bar{y} = 5354.25$ so the coordinates are $(11, 5354.25)$.

3. $\sum_{i=1}^{n} (x_i - \bar{x})^2 = 572$

$$b = \frac{45009}{572} \approx 78{,}69 \text{ and } a = 5353 - 78{,}69 \cdot 11 = 4487{,}41$$

4. $f(40)=78{,}7\cdot 40+4488=7636$
5. Let X be the random variable that models the distribution of salaries.
   $P(X<6459)\approx 0.977$
6. $P(X\geq 7636)\approx 0$ This is consistent with the fact that very few people would have more than 40 years of experience in the company.
7. Let Y be the random variable that models the distribution of salaries in that other company and m its mean.
   $Z=\dfrac{Y-m}{620}$ follow the standard normal distribution. Also,

$$P(Y\geq 5000)\approx 0.107$$
$$P(Y\leq 5000)\approx 0.893$$
$$P\left(\frac{Y-m}{620}\leq\frac{5000-m}{620}\right)\approx 0.893$$
$$P\left(Z\leq\frac{5000-m}{620}\right)\approx 0.893$$

This yields

$$\frac{5000-m}{620}\approx 1.243$$
$$5000-m\approx 770.66$$
$$m\approx 5000-770.66$$
$$m\approx 4229.34$$

So the mean salary in the company is about 4229 €.